Homework 3

Due Date: Monday February 7, 2005.

There is a possible 52 points for this homework assignment.

Problem 1. (18 pts.) Use the pumping lemma to show that each of the following languages is not regular.
1. \( L = \{0^n10^{2n}|n \geq 0\} \)
2. \( L = \{0^i1^j0^k|k > i + j\} \)
3. \( L = \{0^i1^j|i = j \text{ or } j = 2i\} \)
4. \( L = \{0^i1^j|j \text{ is a multiple of } i\} \)
5. \( L = \{x \in \{0,1\}^*|n_0(x) < 2n_1(x)\} \)
6. \( L = \{x \in \{0,1\}^*|\text{no prefix of } x \text{ has more } 1\text{'s than } 0\text{'s}\} \)

Problem 2. (16 pts.) For each statement below, decide whether it is true or false. If it is true, prove it. If not, give a counterexample. All parts refer to languages over the alphabet \( \{0,1\} \).
1. If \( L_1 \subseteq L_2 \) and \( L_1 \) is not regular, then \( L_2 \) is not regular.
2. If \( L_1 \subseteq L_2 \) and \( L_2 \) is not regular, then \( L_1 \) is not regular.
3. If \( L_1 \) and \( L_2 \) are nonregular, then \( L_1 \cup L_2 \) is nonregular.
4. If \( L_1 \) and \( L_2 \) are nonregular, then \( L_1 \cap L_2 \) is nonregular.
5. If \( L \) is nonregular, then \( L' \) is nonregular.
6. If \( L_1 \) is regular and \( L_2 \) is nonregular, then \( L_1 \cup L_2 \) is nonregular.
7. If \( L_1 \) is regular, \( L_2 \) is nonregular, and \( L_1 \cap L_2 \) is regular, then \( L_1 \cup L_2 \) is nonregular.
8. If \( L_1 \) is regular, \( L_2 \) is nonregular, and \( L_1 \cap L_2 \) is nonregular, then \( L_1 \cup L_2 \) is nonregular.

Problem 3. (18 pts.) A number of languages over \( \{0,1\} \) are described below. In each case, decide whether the language is regular or not, and prove your answer is correct.
1. The set of all strings \( x \) beginning with a nonnull string of the form \( ww \).
2. The set of all strings $x$ containing some nonnull substring of the form $ww$.

3. The set of odd-length strings over $\{0, 1\}$ with the middle symbol 0.

4. The set of even-length strings over $\{0, 1\}$ with the two middle symbols equal.

5. The set of string over $\{0, 1\}$ of the form $xyx$ for some $x$ with $|x| \geq 1$.

6. The set of nonpalindromes.