Homework 6

Due Date: Monday March 14, 2005.

There is a possible 46 points for this homework assignment.

Problem 1. (20 pts.) In each case, show using the pumping lemma that the given language is not a CFL.
   a) \( L = \{a^ib^jc^k|i < j < k\} \)
   b) \( L = \{x \in \{a, b\}^*|n_b(x) = n_a(x)^2\} \)
   c) \( L = \{a^n b^{2n} a^n|n \geq 0\} \)
   d) \( L = \{x \in \{a, b, c\}^*|n_a(x) = \max\{n_b(x), n_c(x)\}\} \)
   e) \( L = \{a^n b^m a^n b^{n+m}|m, n \geq 0\} \)

Problem 2. (20 pts.) Decide in each case whether the given language is a CFL, and prove your answer.
   a) \( L = \{a^n b^m a^n b^n|m, n \geq 0\} \)
   b) \( L = \{xayb|x, y \in \{a, b\}^* \text{ and } |x| = |y|\} \)
   c) \( L = \{xcx|x \in \{a, b\}^*\} \)
   d) \( L = \{xyx|x, y \in \{a, b\}^* \text{ and } |x| \geq 1\} \)
   e) \( L = \{x \in \{a, b\}^*|n_a(x) < n_b(x) < 2n_a(x)\} \)

Problem 3. (6 pts.) For these two languages, just provide a Yes (if it is a CFL) or No, it is not answer along with a one sentence justification.
   a) \( L = \{x \in \{a, b\}^*|n_a(x) = 10n_b(x)\} \)
   b) \( L = \) the set of non-balanced parenthesis.