Homework 7

Due Date: Monday, March 20, 2006.

There is a possible 46 points for this homework assignment.

Problem 1. (20 pts.) In each case, show using the pumping lemma that the given language is not a CFL.
   a) \( L = \{a^i b^j c^k | i < j < k \} \)
   b) \( L = \{x \in \{a, b\}^* | n_b(x) = n_a(x)^2 \} \)
   c) \( L = \{a^n b^{2n} a^n | n \geq 0 \} \)
   d) \( L = \{x \in \{a, b, c\}^* | n_a(x) = \max\{n_b(x), n_c(x)\} \} \)
   e) \( L = \{a^n b^m a^n b^{n+m} | m, n \geq 0 \} \)

Problem 2. (20 pts.) Decide in each case whether the given language is a CFL, and prove your answer.
   a) \( L = \{a^n b^m a^m b^n | m, n \geq 0 \} \)
   b) \( L = \{xayb | x, y \in \{a, b\}^* \text{ and } |x| = |y| \} \)
   c) \( L = \{xcx | x \in \{a, b\}^* \} \)
   d) \( L = \{xyx | x \in \{a, b\}^* \text{ and } |x| \geq 1 \} \)
   e) \( L = \{x \in \{a, b\}^* | n_a(x) < n_b(x) < 2n_a(x) \} \)

Problem 3. (6 pts.) For these two languages, just provide a Yes (if it is a CFL) or No, it is not answer along with a one sentence justification.
   a) \( L = \{x \in \{a, b\}^* | n_a(x) = 10n_b(x) \} \)
   b) \( L = \) the set of non-balanced parenthesis.

Problem 4. (5 pts. Extra Credit) Exercise 3.31 on page 115 in the text.

Problem 5. (5 pts. Extra Credit) Exercise 3.32 on page 115 in the text.