CSCI 250 GOLDWEBER
Spring 08 4/14/08

Homework 8

Due Date: Monday April 21, 2008.

There is a possible 47 points for this homework assignment.

Problem 1. (3 pts) Is the following statement true or false? If $L_1, L_2, \ldots$ are recursively enumerable subsets of Σ^*, then $\bigcup_{i=1}^{\infty} L_i$ is recursively enumerable. Give reasons for your answer.

Problem 2. (4 pts.) *Fermat's Last Theorem*, until recently one of the most famous unproved statements in mathematics, asserts that there are no integer solutions (x, y, z, n) to the equation $x^n + y^n = z^n$ satisfying $x, y, z > 0$ and $n \geq 3$. Show how a solution to the halting problem would allow one to determine the truth or falsity of the statement.

Problem 3. (40 pts.) For each decision problem given, determine whether it is solvable or unsolvable, and prove your answer.

a) Given a TM T, does it ever reach a state other than its initial state if it starts with a blank tape?

b) Given a TM T and a non-halting state q of T, does T ever enter state q when it begins with a blank tape?

c) Given a TM T and a non-halting state q of T, is there an input string x that would cause T eventually to enter state q?

d) Given a TM T, does it accept the string ϵ in an even number of moves?

e) Given a TM T, is there a string it accepts in an even number of moves?

f) Given a TM T and a string w, does T loop forever on input w?

g) Given a TM T, are there any input strings on which T loops forever?

h) Given a TM T and a string w, does T reject input w?

i) Given a TM T, are there any input string rejected by T?

j) Given a TM T, does T halt within ten moves on every string?