CSCI 250 GOLDWEBER
Spring 08 4/23/08

Homework 9

Due Date: Wednesday April 30, 2008.

There is a possible 85 points for this homework assignment.

Problem 1. (5 pts. Polynomial reduction) Let $\Sigma = \{a, b\}$, let L_1 be the language of palindromes over Σ, and let $L_2 = \{yy | y \in \Sigma^*\}$. Show that $L_1 \leq L_2$.

Problem 2. (10 pts.) Problem 6.5 on page 195 in the text; parts a and b only.

Problem 3. (10 pts. k-satisfiability) Show that if $k \geq 4$, the k-satisfiability problem is $\mathcal{NP}$-complete.

Problem 4. (15 pts.) Problem 6.6 on page 196 in the text.

Problem 5. (15 pts.) Problem 6.7 on page 196 in the text.

Problem 6. (15 pts.) Consider the following problem:
LONGEST PATH
INSTANCE: Graph $G = (V, E)$, positive integer $k \leq |V|$.
QUESTION: Does G contain a simple path (that is, a path encountering no vertex more than once) with k or more edges?

Prove that **LONGEST PATH** is $\mathcal{NP}$-complete. Using a reduction from **HAMILTONIAN PATH** is highly recommended.

Problem 7. (15 pts.) Consider the following problem:
HITTING SET
INSTANCE: Collection C of subsets of a finite set S, and a positive integer k.
QUESTION: Is there a *hitting set* of k elements from S that includes at least one element from every subset in C?

For example, consider

$$S = \{a, b, c, d, e, f, g\}$$
$$C = \{\{a, b, c\}, \{a, e, g, \}, \{b, e, f, g\}, \{a, d, f\}\}$$
$$k = 2$$

Then the answer is "YES" because the set $\{a, f\}$ hits every subset of C.

Prove that **HITTING SET** is $\mathcal{NP}$-complete. Using a reduction from **VERTEX COVER** is highly recommended.

Problem 8. (Extra Credit 10 pts.) Consider the following problem:
SET SPLITTING
INSTANCE: A collection C of subsets of a finite set S.
QUESTION: Is there a partition of S into two sets S_1 and S_2 such that no subset of C is entirely contained in either S_1 or S_2?

Prove that **SET SPLITTING** is $\mathcal{NP}$-complete. Using a reduction from **3SAT** is highly recommended.

Hint: Given an instance of **3SAT**, f say, that contains n variables start the creation of an instance of **SET SPLITTING** by letting S be the set of all the variables in f and their negations plus one new symbol, a:

$$S = \{a, x_1, \overline{x_1}, x_2, \overline{x_2}, \ldots, x_n, \overline{x_n}\}$$

It remains for you to choose a suitable collection of subsets, C, and to explain why the instance of **SET SPLITTING** created thus has a solution if and only if f is satisfiable.

Problem 9. (Extra Credit 10 pts.) Consider the following problem
DOMINATING SET
INSTANCE: Graph $G = (V, E)$, positive integer $k \leq |V|$.
QUESTION: Is there a subset $V' \subseteq V$ of at most k vertices such that each vertex $V - V'$ (in V but not in V') there is a node $w \in V'$ such that $\{u, w\} \in E$.

Prove that **DOMINATING SET** is $\mathcal{NP}$-complete. Using a reduction from **VERTEX COVER** is highly recommended.