The Halting Problem is Unsolvable

Theorem 1
*The halting problem is unsolvable, i.e. the language **Halting** is not recursive.*
Proof: Proof by contradiction: Assume that the language *Halting* (H) is recursive and is decided by TM T_H s.t. $L(T_H) = H$.

We need a function f s.t. $x \in SA$ if and only if $f(x) \in H$. If one can decide H then one can decide SA (build T_{SA} in terms of T_H) $\Rightarrow$ contradiction.

Let $T_{SA} = T_1 T_H$, where T_1 is a TM with input alphabet $\Sigma = \{x, y\}$ that transforms a tape of the form $\underline{\Delta} w$ to the tape $\underline{\Delta} we(w)$. The construction of T_1 is relatively straightforward and not reproduced here.

If $w \in SA$, then $w = e(T)$ for some TM T, where T accepts w. In this case T_{SA} halts on input w with output 1, since the string $we(w) = e(T)e(w)$ is in *Halting* and T_H decides *Halting*.

If T_{SA} halts with output 1 on input string w, then $we(w) \in$*Halting*. This means that for some TM T and some w, T accepts w. Since $w = e(T), w \in SA$.

If *Halting* were recursive then we can construct the TM T_{SA} which decides SA. Since we know that SA is not recursive, we conclude that neither is *Halting*. $\diamond$