APPENDIX I

CHR. HUYGENS

OEUVRES COMPLÈTE, TOME XIV. P. 92–95

If there remains 1 game to win to A and 1 to B, and 2 games to C, how much will the position be worth of each bettor if they have set each 2 écus into the game?

If C loses the first game he will lose 2 écus, if he wins it he will be clear, but it is two against one that he wins this first game, there are therefore two chances in order to lose 2 écus, and 1 in order to be clear. Hence I say that his part of 6 écus is \( \frac{2}{3} \) écu.\(^1\) Because taking these \( \frac{2}{3} \) écu, he will find two others who will put \( \frac{2}{3} \) écu against his \( \frac{2}{3} \).\(^2\) And [they] will play to who will have the total, namely 2 écus, in what will have same chance as before, namely 2 chances in order to lose his \( \frac{2}{3} \) écus, that is to say 2 écus, adding that which he will have left to A and B: and one chance in order [93] to be clear, by winning from the two others each their \( \frac{2}{3} \) écu. Because thus he will have 2 écus, as before as he had played against A and B. It follows from it that the positions of A and B are worth each \( \frac{2}{3} \) écus. And if we divide that which is in the game into 9 parts, A will take 4 of it. B 4. C 1.\(^3\)

If there remains 1 game to win to A, 2 to B, and 2 to C, 6 écus in the game.

If A wins the first he wins 4 écus. If B or C win it, A wins \( \frac{2}{3} \) écus by the preceding, therefore A has 2 chances in order to win \( \frac{2}{3} \) écus and 1 chance in order to win 4 écus. I say that of the 6 écus his part is \( \frac{3}{7} \) écus. That is to say that he wins \( \frac{1}{7} \) écus, because taking beyond his 2 écus which he had set, still \( \frac{1}{7} \) écus that I say he wins he will set \( \frac{10}{9} \) écus each in it, in order to play who will win all. And thence he will have 1 chance in order to win \( \frac{30}{9} \) écus which with the \( \frac{5}{6} \) écus which he will have set aside, will make him win 4 écus and 2 chances in order to win only \( \frac{5}{9} \) it is \( \frac{2}{3} \) écus which he will have set in part. Therefore B and C will have of the 6 écus each \( \frac{1}{9} \) écus. And if one divides that which is in the game into 27 parts, A will take 17, B and C each 5.\(^4\)

If there remains 1 to A, 2 to B, 3 games to C, 6 écus in the game. How much is the position of each worth.

---

\(^1\)See Prop. III. Huygens, instead of corresponding to this Proposition, follows a demonstration which is independent.

\(^2\)In this manner Huygens makes the demonstration of his solution rest on the axiom which he has announced at the beginning of his Treatise.

\(^3\)Compare the solution of Prop. VIII.

\(^4\)Compare the two cases to the Table for three players.
If A wins the first game he wins 4 écus\(^5\) and one in order to win \(1\frac{2}{3}\) écus\(^6\), which are worth as much as he was \(94\) assured to win (in case of loss of the said first game) \(\frac{11}{9}\) écus, which is worth the half of the said \(\frac{2}{3} + 1\frac{7}{9}\). Now he has two chances to lose the first game, and one chance in order to win it, therefore he has one chance in order to win 4 écus and 2 in order to win \(\frac{11}{9}\) écus. I say that he wins \(2\frac{4}{27}\) or else \(\frac{34}{27}\) écus. Because taking this and setting aside \(\frac{11}{9}\) écus he will set the remaining which is \(\frac{16}{27}\) against two others who will put each as much. And thus he will have 1 chance in order to win \(\frac{25}{27}\) écus that is to say (adding \(\frac{11}{9}\) which he is reserved himself) \(\frac{36}{27}\) or 4 écus and 2 chances in order to win only the \(\frac{11}{9}\) écus which he is reserved himself. His part therefore of the 6 écus is \(2 + 2 + \frac{4}{27}\) écus it is \(\frac{4}{27}\) écus.\(^7\)

In order to know how much will be B. I say, if B wins the first game he will win \(\frac{2}{3}\) écus per primam. If he loses it, he courts equal fortune to lose 2 écus or to lose \(\frac{8}{3}\) écus per secundam, which is as much as if losing this first game he would lose \(\frac{13}{9}\) écus namely the half of \(2 + \frac{8}{3}\). Now there are 2 chances in order to lose the 1st game and 1 chance in order to win it. Therefore there are 2 chances in order to lose \(\frac{12}{3}\) écus and 1 chance in order to win \(\frac{7}{3}\) écus. I say that he will have of the 6 écus \(\frac{34}{27}\) or \(1\frac{7}{27}\) écus.\(^8\) Because taking \(\frac{34}{27}\) écus he will set \(\frac{19}{27}\) écus in it against two others who each will put in it as much and thus will have 1 chance in order to win \(\frac{10}{27}\) écus, which with \(\frac{15}{27}\) écus or \(\frac{5}{9}\) [95] that he has reserved aside are \(\frac{24}{9}\) or \(2\frac{1}{3}\), that is to say that he will win \(\frac{2}{3}\) écus. And two chances in order to have only the \(\frac{5}{6}\) écus you reserve, it is to lose \(\frac{11}{9}\) écus of it because he had put 2 écus into the game.

It follows from it that C will have of the 6 \(\frac{16}{27}\) écus.\(^9\) Therefore if one divides the whole into 81 parts, A will take 56, B 27, C 8.\(^10\)

---

\(^5\)Evidently the question here is of the case where it is B who wins. Then there remains to A 1 game, to B 1 and to C 3 games to win. Huygens should have therefore calculated the “chances” of this case and he would have found \(\frac{8}{3}\) instead of \(\frac{2}{3}\) écu. It is only inadvertently that he has taken this last number which agrees in the case, treated at the end of this piece, where there lacks to A 1 game, to B 1 and to C 2 games.

\(^6\)One reads in the margin “by the preceding;” see, indeed, the solution of the case where there remains to A 1 to B 2 and to C 2 games to win.

\(^7\)In truth \(\frac{4}{27}\).

\(^8\)Same confusion between the cases 1, 1, 2 and 1, 1, 3. One must replace, as before, \(\frac{2}{3}\) by \(\frac{8}{9}\).

\(^9\)In truth \(1\frac{4}{9}\).

\(^10\)In truth \(\frac{3}{2}\).

\(^11\)In the “Table for three players” this erroneous solution is replaced by the correct according to which of 27 parts A will take 19, B 6 and C 2.