ÉCLAIRCISSEMENT
relatif au Mmoire sur la mortalité de la petite vrole
qui se trouve dans le Volume de 1796.*

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Mémoires de l’Académie Royale des Sciences et Belles-Lettres. . . Berlin
1804, Class de math., pp. 80–82

§ 15 of this Memoir contains many faults of impression, which it is important to
observe. At line 13, instead of being divided by 42, put being added to 1300, then
divided by 2.42. At line 22, instead of divided by 36, put added to 887, then divided by
2.36. At line 30, instead of divided by 34.7, put added to 643, then divided by 2.34, 7.
At line 39, instead of divided by 30.4, put added to 506, then divided by 2.30, 4. At
line 48, instead of divided by 44, put added to 403, then divided by 2.44. (I count the
lines from the beginning of the paragraph.)

Of the rest, I must caution that the method of approximation that I have given in
this memoir as a test, by awaiting that some more detailed observations put us in a state
to proceed with more regularity, that this method, I say, is worth absolutely nothing,
& I owe some excuses to the public in order to have presented it to them. Although
this approximation satisfies rather well to the observations, however, if one proceeds
rigorously, one finds some essentially different results. Now, without leaving some
assumptions that I have employed in this approximation, one can proceed rigorously as
one is going to see. I resume in § 11 the formula

\[ b^{(i)} = \frac{a^{(i)} b^{(i-1)} (1 - \frac{1}{n^{(i-1)}})}{a^{(i-1)} - \frac{b^{(i-1)}}{m^{(i-1)} n^{(i-1)}}} \]

Now, let \( g^{(i-1)} \) be the number of deaths of smallpox, the number of those who would
have contracted smallpox will be \( m^{(i-1)} g^{(i-1)} \); subtracting this number from \( b^{(i-1)} \),
one will have \( b^{(i-1)} - m^{(i-1)} g^{(i-1)} \) for the number of persons exempt from smallpox
reported at the beginning of the year. For it to report at the end, following the assump-
tions of the approximation, it is necessary to multiply by \( \frac{a^{(i)}}{a^{(i-1)}} \), this which gives a
second value of \( b^{(i)} \). Matching these two values, one has by dividing by

\[ a^{(i)} \frac{a^{(i)} b^{(i-1)} (1 - \frac{1}{n^{(i-1)}})}{a^{(i-1)} - \frac{b^{(i-1)}}{m^{(i-1)} n^{(i-1)}}} = \frac{b^{(i-1)}}{a^{(i-1)}} - \frac{m^{(i-1)} g^{(i-1)}}{a^{(i-1)}} \]

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whence one deduces

\[ m^{(i-1)}n^{(i-1)} = \frac{a^{(i-1)}b^{(i-1)}m^{(i-1)} + b^{(i-1)}m^{(i-1)}g^{(i-1)} - b^{(i-1)}g^{(i-1)}}{a^{(i-1)}m^{(i-1)}g^{(i-1)}}. \]

Now

\[ b^{(i)} = \frac{a^{(i)}b^{(i-1)} + a^{(i)}m^{(i-1)}g^{(i-1)}}{a^{(i-1)}}, \]

\[ \frac{b^{(i)} + b^{(i-1)}}{2g^{(i-1)}} = m^{(i-1)}n^{(i-1)} = \frac{a^{(i)}b^{(i-1)} + a^{(i)}b^{(i-1)} - a^{(i-1)}m^{(i-1)}g^{(i-1)}}{2a^{(i-1)}g^{(i-1)}}. \]

Matching these two values of \( m^{(i-1)}n^{(i-1)} \) & resolving the equation of the second degree, one finds

\[ m^{(i-1)} = \frac{b^{(i-1)}}{a^{(i)}g^{(i-1)}} \left( \frac{a^{(i-1)} - a^{(i)}}{2} + g^{(i-1)} \right) \left\{ \sqrt{1 + \frac{2a^{(i)}g^{(i-1)}}{(a^{(i-1)} - a^{(i)})^2 + g^{(i-1)}^2}} - 1 \right\}, \]

\[ b^{(i)} = \frac{b^{(i-1)}}{a^{(i-1)}} \left\{ \frac{a^{(i-1)} + a^{(i)}}{2} + g^{(i-1)} - \left( \frac{a^{(i-1)} - a^{(i)}}{2} + g^{(i-1)} \right) \sqrt{1 + \frac{2a^{(i)}g^{(i-1)}}{(a^{(i-1)} - a^{(i)})^2 + g^{(i-1)}^2}} \right\}. \]

Calculating according to these formulas the observations reported in the memoir cited, I have found that the values of \( n \), instead of remaining near the same, vary enormously and more than the values of \( m \). I have prepared some tables of these variations, but I have suppressed them because I have made reflection that all this calculation rested at the 'bottom under some more or less arbitrary assumptions, & that the slightest change in these assumptions gave of all other results. If one makes

\[ m^{(i-1)}n^{(i-1)} = \frac{b^{(i-1)}}{g^{(i-1)}}, \]

\& \[ b^{(i)} = \frac{a^{(i)}}{a^{(i-1)} - g^{(i-1)}} (b^{(i-1)} - m^{(i-1)}g^{(i-1)}), \]

the two formulas are reduced to one, & the calculation falls.