Rates of change

Functions are most useful when they model changing quantities. Let's develop some concepts for measuring that change.

A function $y = f(x)$ is **increasing on the interval** $a \le x \le b$ when

- larger values of x in the interval are always associated with *larger* values of y than are smaller values of x; or
- the graph of f always *rises* (from left to right) over the interval.

The function is **decreasing on the interval** when

- larger values of x in the interval are always associated with *smaller* values of y than are smaller values of x; or
- the graph of f always *falls* (from left to right) over the interval.

If the function output never changes value over the interval $a \le x \le b$, it is said to be **constant** (or **stationary**) over the interval. (Example 3, p. 13)

The Greek letter Δ (delta) is a standard symbol denoting difference, the change in value of a quantity. Thus we write Δx to represent the difference between two values of x: $\Delta x = x_2 - x_1$. Similarly, Δy is the difference between the corresponding values of y. The *relative* change, or **average rate of change**, in both quantities is measured by a **difference quotient**:

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

It describes how much the output quantity changes for each unit increase in the input quantity, and is measured in *output units per input unit.* (Example: p. 14, #4)

Consequently, *if the function $y = f(x)$ is increasing on the interval $a \le x \le b$, then the average rate of change between any two points in the interval is always positive.* Similarly, *if the function $y = f(x)$ is decreasing on the interval $a \le x \le b$, then the average rate of change between any two points in the interval is always negative.*

Graphically, Δy measures the (vertical) rise from one point to another, while Δx measures the (horizontal) run between the points. The average rate of change in the function between these points represents the **slope** of the line between them.