Exponential Functions

Linear functions are used to describe quantities that grow (or decline) at a constant rate. Quadratic functions are used to describe quantities whose rates of growth grow (or decline) at constant rates. But there is a commonly occurring class of phenomena in which the important quantities grow in a manner different from either of these.

Example: Some **population growth models** are based on a simple assumption: the number of births at any given moment in time is determined by the (essentially) fixed percentage of the population representing those females who are giving birth. That is, *population grows at a fixed percentage rate of change*. For instance, Example 2, p. 107, describes how the population of Mexico increased at a constant annual percentage rate of 2.6% per year during the 1980s.

Example: Many financial investment instruments grow via **compound interest** at constant percentage interest rates. See p. 112, #20.

Example: **Radioactive decay** arises when a fixed percentage of the atoms in a substance spontaneously break down into smaller components, releasing significant amounts of energy in the process. See Example 3, p. 107-108.

Numerical investigation of these different kinds of quantities display similar behaviors: an **initial amount** *a* of the quantity grows (or decays) at a fixed **percentage rate of change** *r* with time *t* (initial time corresponding to $t = 0$). Thus,

the amount at time $t = 1$ is $a(1+r)$,
the amount at time $t = 2$ is $[a(1+r)](1+r) = a(1+r)^2$,
the amount at time $t = 3$ is $[a(1+r)^2](1+r) = a(1+r)^3$,

and so on. It follows that the function for the amount present at any time *t* is given by the formula $f(t) = a(1+r)^t$, or, if we put $b = 1+r$, the formula is more simply given as

$$f(t) = ab^t.$$

The number $b = 1+r$ is called the **growth factor** (or **decay factor**); when the quantity grows, *r* is positive and $b > 1$, and when the quantity decays, *r* is negative and $b < 1$.

As the formula shows the input variable *t* in the exponent, the function *f* above is called an **exponential function**. *Exponential functions are used to model phenomena that grow (or decay) at constant percentage rates.*