Power Functions

Two quantities are in **direct proportion** if *one is a constant multiple of the other*; alternatively, one can say that the one quantity **varies directly with** the other. So for instance, the volume V of a sphere is in direct proportion to the cube of its radius r since the volume formula is $V = \frac{4}{3}\pi r^3$; the **constant of proportionality** (or the **constant of variation**) here is the number $\frac{4}{3}\pi$.

Two quantities are in **inverse proportion** if *one is a constant multiple of the reciprocal of the other*; alternatively, one can say that the one quantity **varies inversely with** the other. For instance, the sound intensity from a point source varies inversely with the square of the distance to the source. This is expressed by the acoustic law $I = \dfrac{P}{4\pi r^2}$, where r is distance (in meters), I is intensity (measured in decibels), and P is a constant measuring the power of the sound source (in watts).

[*Example*: p. 393, #17]

A **power function** is one whose formula takes the form $y = f(x) = kx^p$. (We have already studied power functions for which $p = 0$, 1, and 2; these are constant, linear, and quadratic functions.)

Power functions with even powers p are even functions. Those with odd powers p are odd functions. Power functions with positive powers p have domains including all real numbers. They also have y-intercepts equal to 0. Power functions with negative powers p have domains including all numbers x *except* 0 and have *no* y-intercept.

Recall that $x^{-p} = \dfrac{1}{x^p}$ and that $x^{1/p} = \sqrt[p]{x}$.

If $p > 0$ and $k > 0$, then the power function $f(x) = kx^p$ increases to infinity with increasing x values, that is, $y \to \infty$ as $x \to \infty$.

If $p < 0$ and $k > 0$, then $f(x) = kx^p$ decreases to zero with increasing x values, that is, $y \to 0$ as $x \to \infty$.

If $p > 1$ and $k > 0$, then $f(x) = kx^p$ is concave up for positive inputs. If $0 < p < 1$, the function is concave down for positive inputs.

[*Examples*: p. 393-395, #7, 10, 23, 41]

The formula of a power function, $f(x) = kx^p$, contains two parameters, *k* and *p*, just like the formulas for linear and exponential functions. So, as for those kinds of functions, if we know (or are given) two values for the function, we can solve for the parameters and determine the exact formula.

[*Examples*: p. 393-394, #11, 35]