Mathematical representations of games

Matrix form (also called normal form)

- one player is represented by the rows of a matrix, the other by the columns
- each row corresponds to one of the options (strategies) of the row player; each column corresponds to a strategy of the column player
- each cell of the matrix holds the payoffs to the players when each chooses the corresponding strategy

Examples
- matching pennies
- matching nickels
- shell game
Zero-sum, constant-sum, variable-sum games

Note that in these examples the row player’s payoff at each outcome is always the opposite of the column player’s payoff (what one player wins is taken from the other player), that is, their payoffs sum to 0. Such games are called zero-sum games. We need only record the row player’s payoffs in a zero-sum game, since the column player’s payoffs are the negatives of the row player’s.

- matching nickels plus pennies from heaven

If the sum of the payoffs to the players at each outcome is the same value (not necessarily 0), the game is a constant-sum game. Constant-sum games are strategically equivalent to zero-sum games.

Variable-sum games (those that are not constant-sum) require more sophisticated analyses than do constant-sum games.

- nuclear chicken
- dilemma of the Independents
Information in a game

A game is said to have **perfect information** if both players’ choices, and their resulting outcomes, are known to both players before they make their choices. If this is not the case, the game is said to have **imperfect information**.
Game trees (also called extensive form)

- **nodes** represent points in the game where a player makes a decision; nodes are labeled with the name of the player who makes the choice there
- **branches** leading out of a node represent all the choices available there
- tree grows from the **root** node (the first player’s first choice) through various levels of play to the final nodes, called **leaves**, which represent the various endpoints of the game; leaves are additionally labeled with the ultimate payoffs to the players (or if the game is zero-sum, just the payoff to the row player)

**Examples**
- pile of stones
- boxes
- glass-shell game

**From extensive form to normal form**

Each path from root to leaf becomes a cell of the matrix; the strategies that intersect at that cell represent the chain of choices made by the player along the corresponding path.