Inflection points and the second derivative

We have seen that an inflection point occurs where the concavity of a curve changes, from concave up to concave down, or vice versa. But we have also noticed that at an inflection point, the slope of the curve takes a maximum value (if the function is increasing there), or a minimum value if the function is decreasing there. That is, an inflection point occurs at a point of most rapid increase or at a point of most rapid decrease.

Since the rate of increase or decrease of the function $f(x)$ is measured by the derivative $f'(x)$, an inflection point will occur where $f'(x)$ attains a relative extreme value. But we know how to find extreme values of $f'(x)$: look for where its derivative equals 0, i.e., where the second derivative $f''(x)$ — the derivative of the derivative — equals 0: Inflection points occur where the second derivative equals zero.
An inflection point for an increasing function $f(t)$ (with an input variable $t$ of time), that is, a point at which the rate of increase $f'(t)$ is greatest, can be interpreted in context as the point of **diminishing returns**. The function is increasing faster and faster before the inflection point, but is increasing more and more slowly afterwards. This idea is useful in many applied situations.