The following 4 problems are all about understanding what the derivative at a point tells us...

For each part, sketch a possible graph of the function (relationship) which has the properties in (a) and (b), and say in a sentence what the information given in (a) and in (b) means in the given context.

(1) Let \( P(t) \) be the number of people in some deer population, \( t \) years after some deer were released on an uninhabited island.

(a) \( P(20) = 635 \)

(b) The slope of the tangent line to the graph of \( P(t) \) at the point \((20,635)\) is equal to \(-10\).

(2) Let \( D(t) \) be the distance fallen (in feet), \( t \) seconds after a rock has been released from a height of 1000 feet. (So, it can't fall more than 1000 feet.

(a) \( D(5) = 687 \)

(b) \( D'(5) = 50 \).

(3) Let \( P(p) \) be the monthly profit of a small operation in $, when every employee is paid \( p \) $ per hour of work.

(a) \( P(10) = 40,000 \)

(b) \( P'(10) = 2,500 \).

(4) Make up an example like the above, where the input is not time. Try to use numbers which you think are reasonable! State both the information (a) and (b) as I have done above, and then say what these pieces of information mean in your own words.
(5) Suppose that \( C(f) \) is the number of bushels of corn produced on a tract of farmland when \( f \) pounds of fertilizer are used during the planting season.

(a) Is it possible for \( C(f) \) to be negative? If yes, what does that mean?

(b) Is it possible for \( C'(f) \) to be negative? If yes, what does that mean?

(c) What would be the meaning of \( C(1500) = 60,000 \)?

(d) What would be the meaning of \( C'(1000) = 42 \)?

(6) Let \( T(p) \) be the number of tickets from Boston to Washington D.C. that a certain airline sells in one week when the price of the ticket is \( p \) dollars. Interpret the following in a sentence:

(a) \( T(115) = 1750 \)

(b) \( T'(115) = -20 \)

(c) \( \frac{dT}{dp} = -2 \) when \( p = 125 \)

(7) Charlie has decided that it is time to lose weight...
In consultation with his family doctor, and his sister, who is an avid runner and nutritionist by profession, he enters a 15 week diet and exercise regimen. Below are 6 different pieces of information about his weight at different points in time. Draw a possible graph of the function \( W(t) \), where \( t \) is the number of weeks after Charlie started the program, and where \( W \) is his weight in pounds:

1. \( W(0) = 325 \)
2. \( W(12) = 285 \)
3. \( W'(1) = -5 \)
4. \( W'(9) = -2.5 \)
5. \( W'(12) = 0 \)
6. \( W'(14) = 0.5 \)

Before you start, you should do several things:
(a) Decide on which ranges of values you will need on the \( t \)- and \( W \)-axes. Then draw a coordinate system which accommodates these ranges, and label the axes.

(b) You will need to know how steep certain slopes will look... For example, how steep is a line with slope \(-5\)? (sketch one somewhere on the side...
The following table shows the US population in millions for the time period 1790 - 1990:

<table>
<thead>
<tr>
<th>year</th>
<th>1790</th>
<th>1810</th>
<th>1830</th>
<th>1850</th>
<th>1870</th>
<th>1890</th>
<th>1910</th>
<th>1930</th>
<th>1950</th>
<th>1970</th>
<th>1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Pop.</td>
<td>3.9</td>
<td>7.2</td>
<td>12.9</td>
<td>23.2</td>
<td>39.8</td>
<td>62.9</td>
<td>92.0</td>
<td>122.8</td>
<td>150.7</td>
<td>202.2</td>
<td>247.1</td>
</tr>
</tbody>
</table>

(a) Based on the values in the table: How fast did the US population grow between 1810 and 1910? Show your work carefully.

(b) Demonstrate how your work and your result can be seen in the scatterplot of the data, below. You may use words to explain your insight.
(c) How rapidly was the US population growing in the year 1900? Answer in a complete sentence, and show/explain how you got your answer/estimate.

(d) Find a well fitting model that is not unnecessarily complicated. Report your model for the US population. I recommend that you use "years after 1790 as input quantity.

The following part will be HW after the next class:

(e) Next week, we will learn how to get the rate at which the US population was growing at the point labeled 1900 very accurately. We will not necessarily work through this particular example in class - but YOU will then be able to work out the accurate rate for this example as HW.