Table for McDonald employees (in thousands) for the time period 1987-1996:

<table>
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</thead>
<tbody>
<tr>
<td>empl. in 1000's</td>
<td>159</td>
<td>169</td>
<td>176</td>
<td>174</td>
<td>168</td>
<td>166</td>
<td>169</td>
<td>183</td>
<td>212</td>
<td>237</td>
</tr>
</tbody>
</table>

The goal of the exercise is to estimate the rate at which the number of McD employees was growing at the point marked 1994 using calculations.

A well fitting cubic model according to our calculators is

\[ E(t) = 0.507t^3 - 5.271t^2 + 15.429t + 159.506 \]

where \( t \) is years since 1987 and \( E \) is the number of McD's employees in 1000's.

1. Use the graph of the function to estimate the slope of the tangent line at \( t = 7 \) (1994). Interpret this slope in the context as a rate.

(2) In the "Numerical Method" we estimate the rate at which the number of employees is growing at the point marked 1994 \( (t = 7) \), by calculating the average rates between \( t = 7 \) and \( t = 8 \), then between \( t = 7 \) and \( t = 7.1 \), etc. ever shorter time intervals starting at \( t = 7 \) ... (we did this together in class...)

![Graph of McDonald employees](image)
Dr. Rossa introduced the idea of "limit":
If there is something you cannot calculate, the next best thing is to invent an iterative process, in which you calculate numbers which you KNOW approach the number you wanted, but could not calculate. We know that we cannot calculate the slope of the tangent line directly because it is a figment of our imagination, and, in the end, we only know ONE point that is on the tangent line. To calculate a slope of a straight line, we need to know TWO points on the line!
We KNOW that if we zoom in on the graph, we will (eventually) see something straight. We do not know how often we have to zoom, but eventually the tiny piece of the graph we see will look straight. Once it does, the slope of what we see is what we want. If we calculate the slope between the point we zoom in on, and the point on the graph that is at the right edge of the corner of the zooming window, we will not calculate the slope of the tangent line. However, the line of which we get the slope will more and more lie exactly on top of the piece of the graph we see, because that piece of the graph is becoming more and more straight. This is how we know that the slopes we CAN calculate will approach the slope we are looking for.

To do this, we drew windows to envision the process of "zooming" in on the point where \( t = 7 \). The first window showed the portion of the graph between \( t = 7 \) and \( t = 8 \), the next one from \( t = 7 \) to \( t = 7.1 \), the next one from \( t = 7 \) to \( t = 7.01 \), etc.

In each of the windows we calculated the slope of the line connecting the point on the graph for input \( x = 7 \) and the point at the right edge of the window. These slopes were calculated as follows:

- \( x \) from 7 → 8 blue slope: \( \frac{E(8) - E(7)}{1} = 22.043 \)
- \( x \) from 7 → 7.1 blue slope: \( \frac{E(7.1) - E(7)}{.1} = 16.70667 \)
- \( x \) from 7 → 7.01 blue slope: \( \frac{E(7.01) - E(7)}{.01} = 16.2178107 \)
- \( x \) from 7 → 7.001 blue slope: \( \frac{E(7.001) - E(7)}{.001} = 16.16937651 \)
- \( x \) from 7 → 7.0001 blue slope: \( \frac{E(7.0001) - E(7)}{.0001} = 16.1645376 \)
- \( x \) from 7 → 7.00001 blue slope: \( \frac{E(7.00001) - E(7)}{.00001} = 16.164053 \)

Now, take a look at the slopes we were able to calculate. What value do they seem to approach? Are they approaching 16.16? Maybe even 16.164? At least when rounded to three decimal places? It seems like they do.

Notice that using this, admittedly, lengthier method, everyone will come up with the same estimates! This is exactly what we wanted! ... Also, notice, that we still cannot be 100% certain what these values approach! So more work will need to be done. Because we do not just want "more accurate", we really want 100% accurate! And we want to be certain!