

Exam 1 Practice

Problem 1. (3 pts.) What is the relationship, if any, between Modus Tollens and Modus Ponens?

Problem 2. (5 pts.) In Samuel Clemens old house you discover a note in the cupboard about a hidden manuscript. The note contains five true statements about the manuscript's location; where is the manuscript? Don't forget to show your work as to how you determine the manuscript's location.

1. If this house is next to a lake, then the manuscript is not in the kitchen.
2. If the tree in the front yard is an elm, then the manuscript is in the kitchen.
3. This house is next to a lake.
4. The tree in the front yard is an elm or the manuscript is buried under the flagpole.
5. If the tree in the back yard is an oak, then the manuscript is in the garage.

Problem 3. (8 pts.) You know what to do: deduce the conclusion from the premises giving the reason for each step.

$$\begin{aligned} & \sim p \vee q \rightarrow r \\ & s \vee \sim q \\ & \sim t \\ & p \rightarrow t \\ & \sim p \wedge r \rightarrow \sim s \\ \therefore & \sim q \end{aligned}$$

Problem 4. (5 pts.) What is the difference between propositional logic (or calculus) and predicate logic (or calculus)?

Problem 5. (3 pts.) What is a statement? What is a predicate? What is a truth set? How, if at all, are these three terms related?

Problem 6. (4 pts.) Rewrite the following statement, and its negation, using quantifiers and variables:

Somebody loves everybody

Problem 7. (3 pts.) Write the negation of the following statement:

$$\forall x, z \in D, \exists y \in E \text{ such that } (P(x) \vee \sim Q(y)) \Leftrightarrow R(z)$$

Problem 8. (4 pts.) Rewrite the following statement in both English without using the word “necessary,” and using formal notation:

Being on time each day is a necessary condition for keeping this job.

Problem 9. (7 pts.) Prove that the square of any integer has the form $4k$ or $4k + 1$ for some integer k .

Problem 10. (8 pts.) Describe the four methodologies studied for proving (or disproving) the truthiness of a quantified statement.

Problem 11. (5 pts.) Prove that for all integers m and n , if $m + n$ is even then m and n are both even or both odd.

Problem 12. (5 pts.) Prove that for any odd integer

$$\left\lceil \frac{n^2}{4} \right\rceil = \frac{n^2 + 3}{4}$$