MATH 180 GOLDWEBER
Spring 08 4/16/08

Exam 2: Practice

Problem 1. (5 pts.) Prove the following:

$$1 + x + x^2 + x^3 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}$$

Problem 2. (3 pts.) Prove the following property for Fibonnaci numbers is true

$$F(n+3) = 2F(n+1) + F(n)$$

Problem 3. (4 pts.) Provide an explicit formula for *one* of the following two sequences

a. $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 =$

b. $2^2 + 4^2 + 6^2 + \ldots + (2n)^2 =$

For 5 points extra credit, prove your explicit formula is correct.

Problem 4. (4 pts.) Suppose that $f_1, f_2, f_3, \ldots$ is a sequence defined as follows:

$$f_1 = 1 \qquad f_k = 2 \cdot f_{\lfloor k/2 \rfloor} \text{ for all integers } k \geq 2$$

Prove that $f_n \leq n$ for all integers $n \geq 1$

Problem 5. (2 pts.) Transform the following by making the change of variable: $j = i - 1$

$$\sum_{i=3}^{n} \frac{i}{i + n - 1}$$

Problem 6. (8 pts.) Give a recursive definition for the set of all strings of well-balanced parentheses.
Prove that every string in S begins with a "("

Problem 7. (4 pts.) Solve the following recurrence:

$$s_k = -4s_{k-1} - 4s_{k-2}, k \geq 2 \qquad s_0 = 0, s_1 = -1$$

Problem 8. (4 pts.) What is the relationship between recursion and induction?

Problem 9. (7 pts.) Provide an explicit formula the following recurrence:

$$s_k = s_{k-1} + 2k, k \geq 1 \qquad s_0 = 3$$

Use induction to prove correct your conjecture regarding the explicit formula for this recurrence.

Problem 10. (6 pts.) Solve the following recurrence:

$$c_k = c_{k-1} + 6c_{k-2}, k \geq 2 \qquad c_0 = 0, c_1 = 3$$

Problem 11. (3 pts.) Towers of Hanoi with an Adjacency Requirement: Suppose that the monks moving those disk in Hanoi are further restricted to only move a disk from one pole to an adjacent one –in addition to the standard limitations for this problem. Assume poles A and C are at two ends of a row, and pole B is in the middle. Find a recurrence relation for a_k: the minimum number of moves needed to transfer a tower of k disks from pole A to pole C.