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INTERNATIONAL TRANSACTIONS IN OPERATIONAL RESEARCH

IFORS' Operational Research Hall of Fame

John von Neumann

One of the most creative and outstanding mathematicians of the 20th century. Made major contributions to game theory, utility theory, design of stored-program computers, numerical analysis, Monte Carlo simulation.

Born: December 28, 1903, Budapest, Hungary.

Died: February 8, 1957, Washington, DC, USA.

Education: Diploma, Chemical Engineering, Eidgenössische Technische Hochschule, Zürich (1925); Doctorate, Mathematics, University of Budapest (1926).

Key positions: Visiting Lecturer, Princeton University (1930–1933); Professor of Mathematics, Institute for Advanced Study, Princeton (1933–1957); Scientific Advisory Committee, US Army Ballistic Research Laboratory, Aberdeen Proving Ground (1940–1957); Consultant, US Navy Bureau of Ordnance (1941–1955); Consultant, Los Alamos Scientific Laboratory (1943–1955); Member, US Armed Forces Special Weapons Project, Washington, DC (1950–1955); Scientific Advisory Board, US Air Force, Washington, DC (1951–1957); Member, General Advisory Committee, US Atomic Energy Commission, Washington, DC (1952–1954); Commissioner, US Atomic Energy Commission (1955–1957).

Awards and Recognition: Member of National Academy of Sciences; American Academy of Arts and Science; American Philosophical Society; Royal Netherlands Academy of Sciences and Letters, Amsterdam, The Netherlands; Accademia Nazionale dei Lincei, Rome, Italy; Academia Nacional de Ciencias Exactas, Lima, Peru; Instituto Lombardo di Science e Lettere, Milan, Italy. Bôcher Prize, American Mathematical Society (1937); President, American Mathematical Society (1951–1952); Albert Einstein Commemorative Award (1956); Enrico Fermi Award, US Atomic Energy Commission (1956); Medal of Freedom, US Presidential Award (1956).

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On the death of his friend John von Neumann, the physicist Stanislaw Ulam wrote (Ulam 1958): ". . . the world of mathematics lost a most original, penetrating, and versatile mind. Science suffered the loss of a universal intellect and a unique interpreter of mathematics, who could bring the latest (and develop latent) applications of its methods to bear on problems of physics, astronomy, biology, and the new technology." Ulam goes on to discuss von Neumann's contributions to set theory and algebra, theory of functions, measure theory, topology, continuous groups, Hilbert space theory, operator theory, theory of lattices, continuous geometry, theoretical physics, quantum theory, statistical mechanics, hydrodynamics, systems of linear equations and matrix inversion, game theory, economics, theory and practice of electronic computers, Monte Carlo method, theory of automata, probabilistic logic, and nuclear energy and nuclear weapons. Ulam mentions "operational research," more or less in passing, as stemming from von Neumann's and Morgenstern's "classical treatise" *Theory of Games and Economic Behavior* (1994). But, from an operational research (OR) perspective, von Neumann's influence far exceeds a passing glance, as evidenced by the establishment of the prestigious von Neumann Theory Prize by the Operations Research Society of America (ORSA) and the Institute of Management Sciences (TIMS) in 1974, and now sponsored by the Institute of Operations Research and the Management Sciences (INFORMS). In what follows, we recount briefly von Neumann's early years and, with respect to his impact on OR, describe his contributions to game theory, numerical analysis, and Monte Carlo simulation.

John (Jansci, Johnny) von Neumann was born on December 28, 1903 in Budapest, Hungary to Margaret (Kann) and Max Neumann. Max was a successful banker who obtained the appellation Neumann de Margitta (title of nobility) in 1913 from the Emperor Franz Josef. This was later changed by Johnny to the Germanized version von Neumann.

As a young child, Johnny exhibited a photographic memory and a remarkable ability in mathematics. Soon after starting his formal education at the Budapest Lutheran Gymnasium, his mathematics teacher recognized that he was a child prodigy and recommended that he be tutored by a University of Budapest professor, Michael Fekete. A special mathematics program was initiated and by the time Johnny left the Gymnasium, he and Fekete had written and published a joint paper that extended a theorem in analysis (1922). In 1921, von Neumann enrolled in the mathematics program at the University of Budapest, but did not take any classes. He also registered at the University of Berlin where he studied chemistry through 1923. He then moved to Zurich and enrolled in the Eidgenössische Technische Hochschule, where he received an undergraduate degree in chemical engineering in 1925. During his time in Berlin and Zurich, he would return to Budapest at the end of each semester so he could take (and pass) the exams at the University. He was thus able to receive his doctorate in mathematics in 1926.

Von Neumann was appointed Privat Dozent (assistant professor) at the University of Berlin where he remained from 1927 to 1929. He then held the same title at the University of Hamburg through 1930. After spending a semester in 1929 lecturing on quantum mechanics at Princeton University, he was offered a position there as Visiting Professor, which he accepted and held from 1930 to 1933. When the Princeton Institute for Advanced Study was opened in 1933, Johnny was appointed as a Professor in Mathematics, a position he held until his death in 1957. He was the youngest member of the Institute when he joined the newly formed illustrious staff of Albert Einstein, Marston Morse, Oswald Veblen, and Hermann Weyl.

The origins of modern game theory can be traced to the work of Ernst Zermelo and Émile Borel, but it was von Neumann who set the stage for what was to follow by his 1928 Minimax Theorem paper. In it he proved the existence of optimal randomized mixed strategies for any two-person, zero-sum game, as well as the existence of a unique value for the game, the minimax value. There was a long hiatus before von Neumann's next game theory publication in 1944. This came about because of his friendship with the Princeton University economist Oskar Morgenstern who introduced him to the competitive problems inherent in economic activities. Together they wrote the seminal book *Theory of Games and Economic Behavior*. Besides establishing the theory of games in a rigorous fashion, this book also set the stage for the development of modern utility theory by giving it an axiomatic base that leads to an existence theorem of a real-valued utility function (the theorem is proved in the 1947 second edition).

Looking at von Neumann's game theory mathematical results in terms of matrix and linear relationships, one can see how and why von Neumann reacted to George Dantzig's description of the newly formulated linear-programming model when they first met in 1947. Here is that story as told by Dantzig (1982, p. 45):

> On October 3, 1947, I visited him (von Neumann) for the first time at the Institute for Advanced Study at Princeton. I remember trying to describe to von Neumann, as I would to an ordinary mortal, the Air Force problem. I began with the formulation of the linear programming model in terms of activities and items, etc. Von Neumann did something which I believe was uncharacteristic of him. "Get to the point," he said impatiently. Having at times a somewhat low kindling-point, I said to myself "O.K., if he wants a quicky, then that's what he will get." In under one minute I slapped the geometric and algebraic version of the problem on the blackboard. Von Neumann stood up and said "Oh that!" Then for the next hour and a half, he proceeded to give me a lecture on the mathematical theory of linear programs.
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> At one point seeing me sitting there with my eyes popping and my mouth open (after I had searched the literature and found nothing), von Neumann said: "I don't want you to think I am pulling all this out of my sleeve at the spur of the moment like a magician. I have just recently completed a book with Oscar [sic] Morgenstern on the theory of games. What I am doing is conjecturing that the two problems are equivalent. The theory that I am outlining for your problem is an analogue to the one we have developed for games." Thus I learned about Farkas' Lemma, and about duality for the first time.

Thus, in the 1940s, we have the almost simultaneous appearance of modern game theory and the field of linear programming. Both areas have become mainstays of OR theory and its application. But they did not evolve from the exigencies of World War II military operations, as did the origin and practice of early OR. (How this came to be is a story yet to be told.) What is really fascinating and beautiful about these two areas is that, although they were developed independently in two very different research environments, they are intimately related as it can be shown that they solve the same mathematical problem.

Von Neumann is considered to be the originator of modern numerical analysis and a key contributor to the development and application of Monte Carlo simulation. He is also the first one to consider and describe an electronic computer in terms of a logical structure that included the stored-program concept and how such a computer processes information. These three areas – numerical analysis, Monte Carlo simulation, stored-program computers – have had major impacts on the development of OR methods and their application.

Many techniques used in OR require the inversion of matrices (the solution of linear systems), e.g., linear programming and least squares. In the early 1940s, an acceptable matrix inverse was rather difficult to determine because of the matrix size, accuracy in computation, and the human effort required to calculate it. In addition to von Neumann, the computer giants John Atanasoff, Herman Goldstine, and Alan Turing recognized that a stable means of solving $aX = b$ was of great importance. It was the ''. . . absolutely fundamental problem in numerical analysis: how best to solve a large system of linear equations'' (Goldstine, 1972, p. 289).

By the early 1940s, Atanasoff had designed a new ''computing machine for the solution of linear algebraic equations'' that applied Gaussian elimination. He noted:

> The solution of general systems of linear equations with a number of unknowns greater than ten is not often attempted. But this is precisely what is needed to make approximate methods more effective in the solution of practical problems (Goldstine, 1972, p. 124).

The question at that time was whether or not numerical procedures for solving large-scale systems could be developed that would produce accurate solutions. In 1943, a heuristic analysis by the statistician Harold Hotelling indicated that Gaussian elimination was unstable; to achieve a five-digit accuracy for a 100×100 system approximately 65 digits would be needed! This caused Von Neumann and his associates, Valentine Bargmann and Deane Montgomery, to consider iterative procedures in their 1946 report ''Solution of linear systems of high order.'' But von Neumann and Goldstine, figuring that Gauss was too skilled a ''computer'' to be caught in such an accuracy problem, decided to pursue the matter. In their seminal paper (1947), ''Numerical inverting of matrices of high order,'' they concluded that Gaussian elimination was ''very good indeed provided the original problem was not 'ill-conditioned'; in other words, the procedure was stable.'' This paper helped to set the future of modern numerical analysis. According to Goldstine, he and von Neumann were so involved with matrix inversion that Mrs. von Neumann named their newly acquired Irish Setter puppy Inverse.

During and after World War II, von Neumann was involved with the theory and design of nuclear weapons being developed at the Los Alamos Laboratory. He was a recognized expert in quantum theory and hydrodynamics, and, most importantly, he had the rare ability to work with physicists and extract their problems into a mathematical form that could then be subjected to analysis and calculation. One of his associates at Los Alamos was the physicist Stanislaw Ulam. They were both involved in the difficult numerical computations of neutron diffusion and nuclear explosions in general. Ulam traced the birth of the Monte Carlo method to a question that occurred to him while he was playing solitaire during his convalescence from a brain operation in January 1946. Ulam's unpublished account notes:

> . . . the question was what are the chances that a Canfield solitaire laid out with 52 cards will come out successfully? After spending a lot of time trying to estimate them by pure combinatorial calculations, I wondered whether a more practical method . . . might not be to lay it out say one hundred times and simply observe and count the number of successful plays. This was already possible to envisage with the beginning of the new era of fast computers, and I immediately thought of problems of neutron diffusion . . . Later . . . [in 1946, I] described the idea to John von Neumann and we began to plan actual calculations (Eckhardt, 1989, p. 131).

In a letter to Robert Richtmyer (March 11, 1947), theoretical division leader at Los Alamos, von Neumann wrote about the "possibility of using statistical methods to solve neutron diffusion and multiplication problems, in accordance with the principle suggested by Stan Ulam." The name Monte Carlo was coined by the Los Alamos theoretical physicist Nicholas Metropolis, the leader of the group that solved the first computer-based Monte Carlo analysis – the simulation of chain reactions, done on the Aberdeen Proving Ground ENIAC in 1947.

The application of Monte Carlo techniques requires a source of random numbers, either by physical means (counting radiation hits on a Geiger counter) or by arithmetic processes using a generating function. For the latter, von Neumann proposed the middle-square procedure in which the calculations are done by squaring an initial n-digit number (seed) and extracting the middle n digits for the next number in the sequence. Von Neumann recognized that this would yield a pseudo-random sequence, at best; the middle-square method is now out of favor as the sequence can be short, degenerate to a zero, or continuously repeat. Von Neumann cautioned users of arithmetic schemes with the statement (von Neumann, 1951, p. 36):

> Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin. For, as has been pointed out several times, there is no such thing as a random number – there are only methods to produce random numbers, and a strict arithmetic procedure of course is not such a method.

John von Neumann's contributions to mathematics, physics, economics, and computers have helped to define many of the major scientific contributions of the 20th century. His intellect and talents were wide ranging. As measured by his total output, von Neumann's contribution to operations research is just a subset, but what an important subset! Just imagine the scope and breadth and impact that OR would now enjoy if John von Neumann had provided us with his wisdom in a more active and direct manner. He would have filled most of the rooms that adjoin the IFORS Hall of Fame.

Saul I. Gass

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