**The sampling distribution for** \( \bar{x}_1 - \bar{x}_2 \)

We assume that we have a SRS of size \( n_1 \) of independently selected measurements of the quantity \( x_1 \) from a population with population mean value \( \mu_1 \) and standard deviation \( \sigma_1 \), and another SRS of size \( n_2 \) of independently selected measurements of the quantity \( x_2 \) from a population with population mean value \( \mu_2 \) and standard deviation \( \sigma_2 \). Both samples satisfy the 10% Condition as well.

- the mean of the sampling distribution of differences \( \bar{x}_1 - \bar{x}_2 \) is \( \mu_1 - \mu_2 \);

- the variance of the sampling distribution of differences is the sum of the variances of the individual proportion variables; therefore, the standard deviation of the sampling distribution of \( \bar{x}_1 - \bar{x}_2 \) is

\[
SD(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}.
\]

- since the population parameters \( \sigma_1 \) and \( \sigma_2 \) are typically unknown, we estimate the standard deviation with the standard error

\[
SE(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}.
\]
The two-sample $t$-interval

A level $C$ confidence interval for $\bar{x}_1 - \bar{x}_2$ is

\[
(\bar{x}_1 - \bar{x}_2) - t^*_{df} \cdot SE(\bar{x}_1 - \bar{x}_2) \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t^*_{df} \cdot SE(\bar{x}_1 - \bar{x}_2),
\]

or, more compactly,

\[
\mu_1 - \mu_2 = (\bar{x}_1 - \bar{x}_2) \pm t^*_{df} \cdot SE(\bar{x}_1 - \bar{x}_2),
\]

where $t^*_{df}$ is the appropriate critical value of $t$ for the given level of confidence, and its degrees of freedom $df$ is given by the formula

\[
df = \frac{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}{\frac{1}{n_1-1} \left( \frac{s_1^2}{n_1} \right)^2 + \frac{1}{n_2-1} \left( \frac{s_2^2}{n_2} \right)^2} \quad (!!)
\]

Of course, it is best to let the entire confidence interval be computed by your calculator (or other software).

[TI-83: STAT TESTS 2-SampTInt]
The Two-sample \( t \) hypothesis test

- **State hypotheses:**
  Null hypothesis: \( H_0 : \mu_1 = \mu_2 \)
  Alternative hypothesis: \( H_A : \mu_1 > \mu_2; \) or \( \mu_1 < \mu_2; \) or \( \mu_1 \neq \mu_2 \)

- **Choose model:**
  Two independent SRS are selected from nearly normal populations satisfying the 10\% Condition, one of size \( n_1 \) and another of size \( n_2 \), so Student’s \( t \) model with \( df = (n_1 - 1) + (n_2 - 1) \) degrees of freedom applies for the standardized sampling distribution for the statistic

  \[
  t_{df} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}},
  \]

- **Mechanics:**
  Compute \( t_{df} \) statistic; then find \( P \)-value associated with the respective form of \( H_A \):
  \( P = P(T \geq t_{df} \mid H_0) \); or \( P = P(T \leq t_{df} \mid H_0) \); or \( P = 2P(T \geq t_{df} \mid H_0) \).

- **Conclusion:**
  Assess evidence to reject (or fail to reject) \( H_0 \) depending on how small \( P \) is.

[TI-83: STAT TESTS 2-SampTTest]
Tukey’s test for comparing means

• **State hypotheses:**
  Null hypothesis: $H_0 : \mu_1 = \mu_2$
  Alternative hypothesis: $H_A : \mu_1 > \mu_2$ (Note: only one alternative option)

• **Choose model:**
  Two independent SRS are selected from nearly normal populations satisfying the 10% Condition, one of size $n_1$ and another of size $n_2$; the first sample contains the largest of all the measurements and the second contains the smallest.

• **Mechanics:**
  Count how many measurements in the first sample are greater than all measurements in the second, count how many measurements in the second sample are less than all measurements in the first, then add the two counts to obtain the Tukey statistic $\tau$ (‘tau’, a Greek $t$), counting any ties as worth $1/2$.

• **Conclusion:**
  Reject $H_0$ in favor of $H_A$ at $\alpha = 0.05$ if $\tau \geq 7$;
  reject $H_0$ in favor of $H_A$ at $\alpha = 0.01$ if $\tau \geq 10$;
  reject $H_0$ in favor of $H_A$ at $\alpha = 0.001$ if $\tau \geq 13$. 