Properties of Logarithmic Functions

Since \( y = \log x \) is the inverse of the function \( y = 10^x \), and \( y = \ln x \) is the inverse of the function \( y = e^x \), there will be important relationships between the graphs of the exponential functions and their logarithmic inverse functions. As the \( x \)-axis is a horizontal asymptote for the exponential functions, so will the \( y \)-axis be a vertical asymptote for the logarithmic functions. In context, this says that as \( x \) gets smaller (approaches zero), the logarithm of \( x \) becomes more and more negative.

As \( 10^x, e^x \to \infty \) whenever \( x \to \infty \), so we can deduce that \( \log x, \ln x \to \infty \) whenever \( x \to \infty \). In context, this says that logarithms of larger numbers get larger.

We say that a quantity \( A \) is an order of magnitude larger than another quantity \( B \) when \( A \) is 10 times larger than \( B \). That is, \( A = 10B \), or \( A / B = 10 \); in other words, the ratio of the quantities is 10. If \( A \) is three orders of magnitude greater than \( B \), then we mean that \( A = 10 \cdot 10 \cdot 10B = 1000B \), or that \( A / B = 10^3 \). In other words, the order of magnitude is the common logarithm of the ratio of the two quantities.

[Examples: p. 165, #2, 33, 15–17; p. 174, #9, 19]