The Dollar Auction: an example of escalation

The two players, both in possession of a bankroll of size \( b \) (in some unit of currency), make bids in turn to purchase a purse with stakes of size \( s \) (in the same unit of currency). The first player makes a positive bid (not necessarily limited to the size of his bankroll). Each subsequent bid either exceeds the previous bid’s value, or is a pass. The first time a player passes, bidding comes to an end, then both players pay out the value of their highest (most recent) bid to the auctioneer, and the stakes are claimed by the higher bidder.

Note that in this game it is worse to have bid and lost than never to have bid at all.

We assume that the players are rational; that is, they will behave in such a way as to maximize their ultimate payoff in the game, utilizing all the information available to them.

This means that the players will adopt the conservative convention: whenever two bidding options achieve the same resulting payoff, a player will always choose to make the smaller bid so as to maximize his ultimate payoff.
Note the differences in terminology and notational conventions between Hamburger and Taylor when they describe the extensive form of a game:

• For Hamburger, **branches** are defined as the *connections between neighboring nodes* in the game tree. For Taylor, **branches** consist of *paths through the tree* from root to leaf (Taylor calls the leaves **terminal nodes**), consisting of a chain of many individual connections between nodes.

• For Hamburger, **nodes are labeled by the player** who is set to make a decision at that point in the game, and branches are labeled with the various options that the player has to choose from. For Taylor, the lines between nodes are not labeled at all, as players are identified by the depth of the level in the tree; the **nodes carry the labels of the options available** to the player who plays at that level in the game.
Analysis of the dollar auction game attempts to discover how the players ought to play rationally. This is achieved by a method called **backward induction** (or **pruning the tree**).

We first identify all **semiterminal nodes**—nodes which are not terminal, but which are connected *only* to terminal nodes in the next deepest layer of the tree. Assuming rationality, the player who is to move at a semiterminal node will choose that option that maximizes their payoff (and, if there are many choices with the same payoff, which minimizes their bid). It follows that this payoff is the ultimate outcome at that semiterminal node, so we simplify the tree by pruning all the leaves and moving this payoff label to the semiterminal node, *which now becomes a terminal node of the pruned tree*.

Repeat this process until the root node is semiterminal. At this last stage, we will learn what the best option is for the first player and whether he can assure a win in the game by playing this strategy.