Vickrey auctions

Real-world auctions can be classified into two categories,

• **private-value** auctions, in which the object has different value to the various bidders involved (like an auction of an antique lamp), and

• **common-value** auctions, in which the object has the same value to the bidders, but this value is unknown to them all (like the auctioning of oil rights for a plot of land).

Each of these types of auction can be carried out via two different schemes,

• **oral** auctions:
  - **ascending** (English) auctions, in which announced bids escalate until only one bidder remains, or
  - **descending** (Dutch) auctions, in which the auctioneer calls out lower and lower prices until one bidder is willing to pay that price;

• and **sealed-bid** auctions, in which a single, secret bid is made by each bidder (with ties settled by lot):
  - **first-price** auctions call for the high bidder to pay the value of his bid, or
  - **second-price** auctions call for the high bidder to pay the value of the second-highest bid.
William Vickrey, in a 1961 paper, showed that *first-price sealed-bid auctions are strategically equivalent to descending oral auctions*. In both types of auction, bidders must choose how high they should bid to win without knowing how high their opponents will go.

He also showed that *ascending oral auctions are strategically equivalent to second-price sealed-bid auctions*. The winning bidder in an ascending auction is paying just more than what the second-highest bidder can stand to bid. For this reason, second-price sealed-bid auctions are now also called **Vickrey auctions**.

**Vickrey’s Theorem.** In a Vickrey auction, honesty is the best policy: bidding one’s true valuation for the object being sold **weakly dominates** every other bidding strategy (that is, this strategy achieves at least as good an outcome for the bidder as any other strategy might, and is better in at least one scenario).

**Proof.** Let \( v \) represent the true value of the object to the bidder, let \( b \) be the value of the bid he makes, and let \( x \) be the highest bid amongst all his opponents.

Suppose \( x > v \). Bidding anything less than \( x \) loses the auction, a 0 payoff to the bidder, and bidding anything more than \( x \) wins the auction, but results in a negative payoff: the value is \( v \), but the price paid is higher. (The case \( b = x \) is left as an exercise.) So a bid of \( b = v \) results in a payoff at least as good as any other.
Suppose $x < v$. Bidding anything less than $x$ loses the auction, a 0 payoff to the bidder, and bidding anything more than $x$ wins the auction and results in a payoff of $v - x$, the same outcome as for a bid of exactly $v$. (Again, the case of bidding exactly $b = x$ is left as an exercise.) So in this case, a bid of $b = v$ results in a payoff at least as good as any other.

Suppose $x = v$. Bidding exactly $b = v$ results in a tie with at least one other bidder and gives the bidder a chance at winning the object and paying the value of the second highest bid (which is also $v$), for a payoff of 0 in either case. Bidding anything less than $x$ loses the auction for sure, which also ends in a 0 payoff. And bidding anything more than $x$ wins the auction and the object, but results in having to pay $x = v$, which again yields a payoff of 0. Thus, there is no better outcome than bidding exactly $b = v$.

So a bid of $b = v$ is at least as good as any other bid, and is sometimes better. //