Desirable properties of social choice procedures

We now outline a number of properties that are desirable for these social choice procedures:

1. **Pareto**
   
   [named for noted economist Vilfredo Pareto (1848-1923)] If alternative $a$ is preferred to alternative $b$ on every voter’s ballot, then $b$ is never a winner.

2. **Condorcet**
   
   [named for another economist, Marie-Jean-Antoine-Nicolas de Caritat, the Marquis de Condorcet (1743-1794)] If in a pairwise comparison with each of the other alternatives, alternative $a$ emerges the winner (whereby it is referred to as the Condorcet winner), then $a$ is the only winner of the election.

3. **Monotonicity (or non-perversity)**
   
   Suppose alternative $a$ is one of the winners. If one of the voters then modifies his ballot by moving $a$ higher up in his preference ordering, and the election is redecided, $a$ is still declared a winner.

4. **Independence of irrelevant alternatives**
   
   Suppose $a$ is a winner and $b$ is not. If some of the voters then modify their ballots in such a way that their prior preference of $a$ over $b$ (or of $b$ over $a$) is not altered in the process, then when the election is redecided, $b$ is still not declared a winner.
Which of the five social choice procedures satisfy these desirable properties? The following table gives the answers:

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Pareto</th>
<th>Condorcet</th>
<th>Monotonicity</th>
<th>I.I.A.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plurality</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Borda</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Hare</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Seq. Pairs</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Dictator</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

For arguments that show why the cells marked Yes are true, see Taylor, pp. 108–113.

For examples that illustrate why the cells marked No are true, see Taylor, pp. 113–120.

In particular, it is important to note that none of the social choice procedures satisfy all four of the desirable properties. This is not because of some flaw in each one of these procedures; indeed, it is a flaw in the inherent nature of social choice.

**Theorem.** No social choice procedure for dealing with more than two alternatives can satisfy both the Condorcet criterion and the Independence of irrelevant alternatives criterion.
Before we prove this theorem, let us first consider a phenomenon known as the **Condorcet voting paradox** (attributed, of course, to the Marquis de Condorcet). It illustrates some of the difficulties that any social choice function must deal with.

Suppose that a public with three voters faces an election amongst three alternatives \(a, b, c\). If the three ballots are

\[
\begin{array}{ccc}
 p_1 & p_2 & p_3 \\
 a & c & b \\
 b & a & c \\
 c & b & a \\
\end{array}
\]

then which alternative should a “good” social choice procedure choose for a winner?

If \(a\) is chosen the winner, then \(p_2\) and \(p_3\) could argue that they form a majority of the public who prefer \(c\) to \(a\). If \(b\) is chosen the winner, then \(p_1\) and \(p_2\) could argue that they form a majority of the public who prefer \(a\) to \(b\). And if \(c\) is chosen the winner, then \(p_1\) and \(p_3\) could argue that they form a majority of the public who prefer \(b\) to \(c\). Thus no social procedure function can settle on a unique winner that isn’t inferior to some other alternative to a majority of voters!
A proof of the theorem we stated earlier proceeds as follows:

Proof. Suppose there were some social choice procedure that did satisfy both the Condorcet criterion and the Independence of irrelevant alternatives criterion. How then would this procedure deal with the set of ballots

$$\begin{array}{ccc}
p_1 & p_2 & p_3 \\
a & c & b \\
b & a & c \\
c & b & a
\end{array}$$

which we looked at earlier?

Might $a$ be declared a winner? Suppose this is the case and consider what happens if the third voter then switches the positions of alternatives $b$ and $c$. In this event, $c$ would defeat both $a$ and $b$ in a pairwise comparison, hence $c$ would have to be the unique winner, as we are assuming that our procedure satisfies the Condorcet criterion. But in the original profile, the preference of $c$ over $a$ is the same as in the modified profile. Thus, by the I.I.A. criterion, $a$ could not be declared the winner for the original profile of ballots.
Now the (original) profile is symmetric with respect to the three alternatives, so the argument we have given to show that a cannot be a winner can be modified, simply by permuting the alternatives, to show that neither b nor c can be winners.

But then the social choice procedure we are considering has no winners, which is impossible. This contradiction implies that our assumption that the social procedure satisfied both the Condorcet criterion and the Independence of irrelevant alternatives criterion could not hold, completing the proof. //