Computing derivatives numerically

Recall that the derivative \( f'(x_0) \) measures the slope of the tangent line to the graph of the function \( f(x) \) at \( x = x_0 \), and that this instantaneous rate of change can be approximated by choosing an input value \( x = x_1 \) very close to \( x_0 \) and computing the average rate of change between these inputs (i.e., the slope of the tangent line is approximated by the slope of a nearby secant line):

\[
f'(x_0) \approx \frac{f(x_1) - f(x_0)}{x_1 - x_0}
\]

for a choice of \( x_1 \) close to \( x_0 \).

By choosing a succession of inputs \( x_1, x_2, x_3, \ldots \) increasingly closer to \( x_0 \), the approximation of \( f'(x_0) \) improves.

Once we reach a point at which the number of significant digits in our improved approximations stabilize to a degree sufficient for our purposes, we can be confident that we have a sufficiently accurate computation of \( f'(x_0) \).

[TI-83: With \( f(x) \) in \( Y_1 \), put \( \frac{Y_1(X) - Y_1(x_0)}{X - x_0} \) into \( Y_2 \), build a table for values of \( X \) closer and closer to \( x_0 \).]