Rules for computing derivative formulas

The method for computing derivative formulas

\[ f\[x] = \lim_{h \to 0} \frac{f(x + h) \cdot f(x)}{h} \]

becomes a very powerful tool when we apply it in certain general cases:

- **Sum Rule**: Say we know the formulas for the derivatives \( g\[x] \) and \( G\[x] \) of the functions \( g(x) \) and \( G(x) \). Then the formula for the function which is the sum of these functions, \( f(x) = g(x) + G(x) \), is

\[ f\[x] = \lim_{h \to 0} \frac{f(x + h) \cdot f(x)}{h} \]

\[ = \lim_{h \to 0} \frac{[g(x + h) + G(x + h)] \cdot [g(x) + G(x)]}{h} \]

\[ = \lim_{h \to 0} \frac{[g(x + h) \cdot g(x)] + [G(x + h) \cdot G(x)]}{h} \]

\[ = \lim_{h \to 0} \frac{g(x + h) \cdot g(x)}{h} + \frac{G(x + h) \cdot G(x)}{h} \]

\[ = g\[x] + G\[x] \]
• **Difference Rule:** A similar kind of computation shows that if \( f(x) = g(x) \square G(x) \), then

\[
f'(x) = g'(x) \square G'(x).
\]

• **Constant Multiplier Rule:** Say we know the formula for the derivative \( g'(x) \) of the function \( g(x) \). Then the formula for a function which is a constant multiple of this function, \( f(x) = k \cdot g(x) \), is

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = \lim_{h \to 0} \frac{k \cdot g(x + h) - k \cdot g(x)}{h} = k \cdot g'(x)
\]

• **Derivative Power Rule:** If \( n \) is a non-zero number and \( f(x) = x^n \), then \( f'(x) = nx^{n-1} \).

These rules make it possible to quickly calculate derivatives for all polynomial functions. This is the “calculus” that Leibniz discovered: procedures for quickly calculating rate-of-change functions.