Inflection points and the second derivative

We have seen that an inflection point occurs where the concavity of a curve changes, from concave up to concave down, or vice versa. But we have also noticed that at an inflection point, the slope of the curve takes a maximum value (if the function is increasing there), or a minimum value if the function is decreasing there. That is, an inflection point occurs at a point of most rapid increase or at a point of most rapid decrease.

Since the rate of increase or decrease of the function $f(x)$ is measured by the derivative $f'(x)$, an inflection point will occur where $f'(x)$ attains a relative extreme value. But we know how to find extreme values of $f'(x)$: look for where its derivative equals 0, i.e., where the second derivative $f''(x)$ — the derivative of the derivative — equals 0: Inflection points occur where the second derivative equals zero.
An inflection point for an increasing function $f(t)$ (with an input variable $t$ of time), that is, a point at which the rate of increase $f'[t]$ is greatest, can be interpreted in context as the point of diminishing returns. The function is increasing faster and faster before the inflection point, but is increasing more and more slowly afterwards. This idea is useful in many applied situations.