3.1 THE GEOMETRY OF LINES

E-1 An alternative version of similarity: We know from Euclidean geometry theorem 2 that if $\triangle ABC$ is similar to $\triangle A'B'C'$ then

$$\frac{|AB|}{|A'B'|} = \frac{|AC|}{|A'C'|}.$$  

Multiplying both sides by $|A'B'|$ and dividing both sides by $|AC|$ gives

$$\frac{|AB|}{|AC|} = \frac{|A'B'|}{|A'C'|},$$

as desired.

E-2 Two-angle criterion for similarity: Let the angles of the first triangle have measure $\alpha$, $\beta$, and $\gamma$, and let the angles of the other triangle have measure $\alpha'$, $\beta'$, and $\gamma'$, chosen so that the two congruence relations correspond to $\alpha = \alpha'$ and $\beta = \beta'$. (We measure all angles in degrees.) We want to show that $\gamma = \gamma'$. Now by Euclidean geometry theorem 1 we have

$$\alpha + \beta + \gamma = 180$$

and

$$\alpha' + \beta' + \gamma' = 180.$$  

Solving for $\gamma$ in the first equation gives

$$\gamma = 180 - \alpha - \beta,$$

and solving for $\gamma'$ in the second gives

$$\gamma' = 180 - \alpha' - \beta'.$$  

Because $\alpha = \alpha'$ and $\beta = \beta'$, we find

$$\gamma = 180 - \alpha - \beta = 180 - \alpha' - \beta' = \gamma'.$$
Thus $\gamma = \gamma'$. Hence the two triangles have congruent angles, and they are similar, as desired.

**E-3 Calculating with similarity**: By Euclidean geometry theorem 2 we have $\frac{10}{5} = \frac{a}{6}$, and multiplying both sides by 6 gives $a = 12$. Applying the theorem again gives $\frac{5}{10} = \frac{b}{6}$, and multiplying both sides by 6 gives $b = 3$.

**E-4 Defining the sine function**: First we show that the triangles $\Delta CAB$ and $\Delta EAD$ are similar. In fact, they share the angle $A$, and both have a right angle, so by the two-angle criterion in Exercise E-2 the triangles are similar. Now we use Euclidean geometry theorem 2:

$$\frac{|BC|}{|DE|} = \frac{|AC|}{|AE|}$$

Multiplying both sides by $|DE|$ and dividing both sides by $|AC|$ gives

$$\frac{|BC|}{|AC|} = \frac{|DE|}{|AE|},$$

as desired.

**E-5 The Pythagorean theorem**:

(a) First we show that the triangles $\Delta ABC$ and $\Delta DAC$ are similar. In fact, they share an angle at $C$, and both have a right angle, so by the two-angle criterion in Exercise E-2 the triangles are similar. Next we show that the triangles $\Delta ABC$ and $\Delta DBA$ are similar. In fact, they share an angle at $B$, and both have a right angle, so by the two-angle criterion in Exercise E-2 the triangles are similar.

(b) Because $\Delta ABC$ and $\Delta DAC$ are similar, by Euclidean geometry theorem 2 we have

$$\frac{|BC|}{|AC|} = \frac{|AC|}{|DC|},$$

which says that

$$\frac{c}{a} = \frac{a}{d}.$$  

Multiplying both sides by $ad$ gives $cd = a^2$, as desired.

(c) Because $\Delta ABC$ and $\Delta DBA$ are similar, by Euclidean geometry theorem 2 we have

$$\frac{|BC|}{|BA|} = \frac{|AB|}{|DB|},$$

which says that

$$\frac{c}{b} = \frac{b}{e}.$$  

Multiplying both sides by $be$ gives $ce = b^2$, as desired.
(d) By Parts (b) and (c) we have the equations $cd = a^2$ and $ce = b^2$. Adding these equations gives

$$a^2 + b^2 = cd + ce.$$ 

Now $d + e = c$ because the length of the line segment $BC$ is the sum of the lengths of $DC$ and $BD$. Thus

$$cd + ce = c(d + e) = c(c) = c^2.$$ 

Combining the last two displayed equations yields $a^2 + b^2 = c^2$, as desired.

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E-6 **Another proof of the Pythagorean theorem**: Behold!

![Diagram of squares and triangles](image)

The two squares in Figure 3.29 have total area $a^2 + b^2$, and (as noted in the text) the square in Figure 3.28 has area $c^2$. This rearrangement shows that $a^2 + b^2 = c^2$.

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E-7 **A right triangle**: First we find the slope of the line passing through $(1, 1)$ and $(3, 4)$. We have

$$\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{4 - 1}{3 - 1} = \frac{3}{2}.$$ 

Next we find the slope of the line passing through $(1, 1)$ and $(4, -1)$. We have

$$\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{-1 - 1}{4 - 1} = \frac{-2}{3}.$$ 

Because the slope $-\frac{2}{3}$ is the negative reciprocal of the slope $\frac{3}{2}$, the two lines are perpendicular. Hence the given points do form the vertices of a right triangle.
Parallel and perpendicular lines:

(a) Solving the first equation for $y$ gives $y = \frac{-3}{2}x + 2$, so the slope of the first line is $\frac{-3}{2}$. Solving the second equation for $y$ gives $y = \frac{-3}{2}x + \frac{9}{4}$, so the slope of the second line is also $\frac{-3}{2}$. Because the slopes are the same, the lines are parallel.

(b) Solving the first equation for $y$ gives $y = \frac{5}{7}x - \frac{3}{7}$, so the slope of the first line is $\frac{5}{7}$. Solving the second equation for $y$ gives $y = \frac{4}{3}x - \frac{8}{3}$, so the slope of the second line is $\frac{4}{3}$. Because the slopes are not the same, the lines are not parallel. Because the slopes are not negative reciprocals of each other, the lines are not perpendicular. Hence the lines are neither parallel nor perpendicular.

(c) Solving the first equation for $y$ gives $y = \frac{-5}{7}x + \frac{15}{7}$, so the slope of the first line is $\frac{-5}{7}$. Solving the second equation for $y$ gives $y = \frac{7}{5}x + \frac{7}{15}$, so the slope of the second line is $\frac{7}{5}$. Because the slopes are negative reciprocals of each other, the lines are perpendicular.

(d) Solving the first equation for $y$ gives $y = \frac{-a}{b}x + \frac{c}{b}$, so the slope of the first line is $\frac{-a}{b}$. Solving the second equation for $y$ gives $y = \frac{-a}{b}x + \frac{d}{bk}$, so the slope of the second line is also $\frac{-a}{b}$. Because the slopes are the same, the lines are parallel.

A system of equations with no solution: Solving the first equation for $y$ gives $y = \frac{-2}{7}x + \frac{9}{7}$, so the slope of the first line is $\frac{-2}{7}$. Solving the second equation for $y$ gives $y = \frac{-2}{7}x + \frac{5}{14}$, so the slope of the second line is also $\frac{-2}{7}$. Because the slopes are the same, the lines are parallel. Evidently, though, the lines have different vertical intercepts, so they are not coincident (that is, not the same). Thus the graphs represented by the two linear equations do not cross. Hence the system has no solution.

Slope from rise and run: We have Slope = $\frac{\text{Rise}}{\text{Run}}$. In this case, the rise is 8 feet, the height of the wall, and the run is 2 feet, since that is the horizontal distance from the wall. Thus the slope is $\frac{8}{2} = 4$ feet per foot.

S-2. Slope from rise and run: We have Slope = $\frac{\text{Rise}}{\text{Run}}$. In this case, the rise is 15 feet, the height of the wall, and the run is 3 feet, since that is the horizontal distance from the wall. Thus the slope is $\frac{15}{3} = 5$ feet per foot.
S-3. **Height from slope and horizontal distance**: We have

\[ \text{Vertical change} = \text{Slope} \times \text{Horizontal change}. \]

In this case, the slope of the ladder is 2.5, while the horizontal distance is 3 feet, so the vertical height is \( 2.5 \times 3 = 7.5 \) feet.

S-4. **Height from slope and horizontal distance**: We have

\[ \text{Vertical change} = \text{Slope} \times \text{Horizontal change}. \]

In this case, the slope of the ladder is 1.7, while the horizontal distance is 4 feet, so the vertical height is \( 1.7 \times 4 = 6.8 \) feet.

S-5. **Horizontal distance from height and slope**: We have

\[ \text{Vertical change} = \text{Slope} \times \text{Horizontal change}. \]

In this case, the slope of the ladder is 1.75, while the vertical distance is 9 feet, so we have \( 9 = 1.75 \times \text{Horizontal change} \). Thus the horizontal distance is \( \frac{9}{1.75} = 5.14 \) feet.

S-6. **Horizontal distance from height and slope**: We have

\[ \text{Vertical change} = \text{Slope} \times \text{Horizontal change}. \]

In this case, the slope of the ladder is 2.1, while the vertical distance is 12 feet, so we have \( 12 = 2.1 \times \text{Horizontal change} \). Thus the horizontal distance is \( \frac{12}{2.1} = 5.71 \) feet.

S-7. **Slope from two points**: We have \( \text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} \). In this case, the vertical change is \(-2\) feet (12 feet dropped to 10 feet) while the horizontal change is 3 feet (since west is the positive direction). Thus the slope is \( \frac{-2}{3} = -\frac{2}{3} \) foot per foot.

S-8. **Continuation of Exercise S-7**: We have \( \text{Vertical change} = \text{Slope} \times \text{Horizontal change} \).

In this case, the slope is \(-\frac{2}{3}\), while the horizontal change is 5, since I move 5 additional feet west. Thus the vertical change is \( -\frac{2}{3} \times 5 = -3.33 \) feet, and so the height of the roof is now \( 10 - 3.33 = 6.67 \) feet.

S-9. **A circus tent**: We have \( \text{Vertical change} = \text{Slope} \times \text{Horizontal change} \). In this case, the slope is \(-0.8\), while the horizontal change is 7 feet, so the vertical change is \(-0.8 \times 7 = -5.6\) feet. Thus the height of the tent if I walk 7 feet west is \( 22 - 5.6 = 16.4 \) feet.
S-10. **More on the circus tent**: We have Vertical change = Slope × Horizontal change. In this case, the slope is $-0.8$ and the vertical change is $-22$ feet (22 feet dropped to 0 feet, for a change of $-22$ feet), so we have $-22 = -0.8 \times$ Horizontal change. Thus the horizontal change is $\frac{-22}{-0.8} = 27.5$ feet. Thus it is 27.5 feet from the center of the tent to where the roof meets the ground.

S-11. **Slope**: We have Slope = $\frac{\text{Rise}}{\text{Run}}$. In this case, the rise is 100 feet, the height of the building, and the run is 70 feet, since that is the horizontal distance to the building. Thus the slope is $\frac{100}{70} = 1.43$ feet per foot.

S-12. **Vertical distance from length and slope**: We have

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change}.
\]

In this case, the slope of the line is 1.8, while the horizontal distance is 130 feet, so the vertical height is $1.8 \times 130 = 234$ feet.

1. **A line with given intercepts**: The picture is drawn below. Since the line is falling from left to right, we expect the slope of the line to be negative. The slope is

\[
m = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{0 - 4}{3 - 0} = \frac{-4}{3}.
\]

![Graph](image)

2. **A line with given vertical intercept and slope**: To make the graph, we start with the vertical intercept at 3 and draw a line with slope 1. That is, the line should rise one unit for each unit of run. Our graph is shown below.

To find the horizontal intercept, we need to know how far to the left we should move to make the graph fall 3 units. Since the graph rises by 1 unit for each unit of run, we need to move 3 units to the left to get to the horizontal intercept. Thus the horizontal intercept is at $-3$. 

![Graph](image)
3. **Another line with given vertical intercept and slope**: Because the graph falls by 2 units for each unit of run, we need to move \( \frac{8}{2} = 4 \) units to the right of the vertical axis to get to the horizontal intercept. Thus the horizontal intercept is 4. More formally, if we move from the location of the horizontal intercept to that of the vertical, we have

\[
\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Vertical intercept} - 0}{0 - \text{Horizontal intercept}},
\]

so

\[-2 = \frac{8}{- \text{Horizontal intercept}},\]

and solving gives the value of 4 for the horizontal intercept.

4. **A line with given horizontal intercept and slope**: The graph rises by 3 units for each unit of run, and we need to move 6 units to the left of the horizontal intercept to get to the vertical axis. Thus the vertical intercept is \(-6 \times 3 = -18\). More formally, if we move from the location of the horizontal intercept to that of the vertical, we have

\[
\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Vertical intercept} - 0}{0 - \text{Horizontal intercept}},
\]

so

\[3 = \frac{\text{Vertical intercept}}{-6},\]

and multiplying both sides by \(-6\) gives the value of \(-18\) for the vertical intercept.

5. **Lines with the same slope**: The first line should have a vertical intercept at 3 and rise 2 units for each unit of run. The second should have a vertical intercept of 1 and rise 2 units for each unit of run. Our picture is shown below. The lines do not cross. Different lines with the same slope are parallel.
6. **Where lines with different slopes meet**: One line should cross the vertical axis at 2 and rise 3 units for each unit of run. The other should cross the vertical axis at 4 and rise 1 unit for each unit of run. Our picture is shown below. These two lines cross to the right of the vertical axis. For the two lines to cross to the right of the vertical axis, the slope of the line with the lower vertical intercept must be greater than the slope of the line with the higher vertical intercept.

![Graph showing two lines crossing to the right of the vertical axis.](image)

7. **A ramp to a building**:
   
   (a) The slope of the graph is 0.4, so one foot of run results in 0.4 foot of rise. Thus, one foot from the base of the ramp, it is 0.4 foot high.
   
   (b) To get to the steps we have moved horizontal 15 feet from the base of the ramp. Since the slope is 0.4, the ramp rises
   
   $$\text{Rise} = \text{Run} \times \text{Slope} = 15 \times 0.4 = 6 \text{ feet}.$$

8. **A wheelchair service ramp**:
   
   (a) Since the slope of the ramp has to be $\frac{1}{12}$, we know that for every horizontal foot we move, we rise $\frac{1}{12}$ vertical foot. The rise we need is 2 feet, or 24 increments of $\frac{1}{12}$. That means we need to start the ramp 24 feet away from the steps. Alternatively, we can do this with a formula:
   
   $$\begin{align*}
   \text{Rise} &= \text{Run} \times \text{Slope} \\
   2 &= \text{Run} \times \frac{1}{12} \\
   24 &= \text{Run} \quad \text{Multiply both sides by 12.}
   \end{align*}$$

   (b) One foot of run allows $\frac{1}{12}$ foot of rise. That is one inch. Thus the allowable rise is one inch for 1 foot of run.

9. **A cathedral ceiling**:
   
   (a) When we move 3 feet east from the west wall, the ceiling rises $10.5 - 8 = 2.5$. So the slope is
   
   $$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{2.5}{3} = 0.83 \text{ foot per foot.}$$
(b) If we move 17 feet to the right of the west wall then the vertical change is

\[
\text{Rise} = \text{Slope} \times \text{Run} = 0.83 \times 17 = 14.11 \text{ feet.}
\]

So the height of the ceiling at that point is \(8 + 14.11 = 22.11 \text{ feet.}\)

(c) Since we start at 8 feet high on the west wall and we want to reach a 12-foot height, then the vertical change is 4 feet. Now

\[
\begin{align*}
\text{Vertical change} &= \text{Horizontal change} \times \text{Slope} \\
4 &= \text{Horizontal change} \times 0.83 \\
\frac{4}{0.83} &= \text{Horizontal change} \\
4.82 &= \text{Horizontal change.}
\end{align*}
\]

So if we place the light 4.82 feet from the west wall, then we will still be able to change the bulb.

10. **Roof trusses**: In both parts we focus on the part of the roof line sloping upward from left to right, and we take movement toward the center as going in the positive direction. Moving along this line from the outer wall to the peak, we have a horizontal change of 8 feet (half the length of the joist) and a vertical change of 4 feet. Thus the slope of this line is \(\frac{4}{8} = 0.5 \text{ foot per foot.}\)

(a) Moving along the line from the outer wall to the vertical strut, we have a horizontal change of 3 feet, and the vertical change is the length of the strut. Because the slope is 0.5, from

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change},
\]

we find

\[
\text{Vertical change} = 0.5 \times 3 = 1.5 \text{ feet.}
\]

Thus the length of the strut is 1.5 feet.

(b) Moving from the tip of the rafter to the top of the wall, we have a horizontal change of 1.5 feet. Because the slope is 0.5, from

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change},
\]

we find

\[
\text{Vertical change} = 0.5 \times 1.5 = 0.75 \text{ foot.}
\]

Thus the outside tip is \(8 - 0.75 = 7.25 \text{ feet above the floor.}\)
11. **Cutting plywood siding:** The shape of the first piece shows that for a horizontal change (to the left) of 4 feet the vertical change along the roof line is $2.5 - 1 = 1.5$ feet. Now each piece has width 4 feet, so we observe that for each piece the longer side and the shorter side differ in length by 1.5 feet.

(a) By the observation above, the length $h$ is $2.5 + 1.5 = 4$ feet.

(b) By the observation above, the length $k$ is $h + 1.5 = 4 + 1.5 = 5.5$ feet, or 5 feet 6 inches.

*Alternative approach:* In both parts we focus on the part of the roof line sloping upward from right to left, and we take movement toward the center as going in the positive direction. Moving along this line from the outer wall to the far corner of the first piece of siding, we have a horizontal change of 4 feet and a vertical change of $2.5 - 1 = 1.5$ feet. Thus the slope of this line is $\frac{1.5}{4} = 0.375$ foot per foot.

For Part(a): Moving along upper edge of the second piece of siding toward the peak, we have a horizontal change of 4 feet. Because the slope is 0.375, from

$$\text{Vertical change} = \text{Slope} \times \text{Horizontal change},$$

we find

$$\text{Vertical change} = 0.375 \times 4 = 1.5 \text{ feet}.$$

To find $h$ we add this vertical change to the length 2.5 feet of the short side of the second piece. Thus the length $h$ is $2.5 + 1.5 = 4$ feet.

For Part(b): Moving along upper edge of the third piece of siding toward the peak, we have a horizontal change of 4 feet. Because the slope is 0.375, from

$$\text{Vertical change} = \text{Slope} \times \text{Horizontal change},$$

we again find

$$\text{Vertical change} = 0.375 \times 4 = 1.5 \text{ feet}.$$

To find $k$ we add this vertical change to the length 4 feet of the short side of the third piece. Thus the length $k$ is $4 + 1.5 = 5.5$ feet.

12. **An overflow pipeline:** Because a 12-foot stretch represents $\frac{12}{96} = \frac{1}{8}$ of the entire horizontal distance of 96 feet, over a 12-foot horizontal stretch the pipe drops by $5 \times \frac{1}{8} = \frac{5}{8}$ foot, or about 0.63 foot. An alternative approach is to compute the slope as $\frac{5}{96}$ (with the proper choice of direction) and to conclude that the vertical change is $\frac{5}{96} \times 12 = \frac{5}{8}$ foot.
13. **Looking over a wall:** Assuming that the man can just see the top of the building over the wall, we focus on the line of sight from the man to the top of the building. Taking south as the positive direction, we compute the slope of the line by considering the change from the top of the wall to the top of the building. We find that the slope of the line is

\[
\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{50 - 35}{20} = \frac{15}{20} = 0.75 \text{ foot per foot.}
\]

Now we find the horizontal distance from the wall to the man by considering the change along the line from the man to the top of the wall. We use

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change},
\]

which says

\[
35 - 6 = 0.75 \times \text{Horizontal distance}.
\]

Dividing both sides by 0.75 shows that the horizontal distance is \(\frac{35 - 6}{0.75} = 38.67\) feet. Thus the man must be at least 38.67 feet north of the wall.

14. **The Mississippi River:**

(a) Since the Gulf of Mexico is at sea level, or 0 feet above sea level, then the vertical change from Lake Itasca to the Gulf of Mexico is \(0 - 1475 = -1475\) feet. Note that the change is negative, since the river is decreasing in elevation as it flows south.

The horizontal change is 2340 miles. So the slope is

\[
\frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{-1475}{2340} = -0.63 \text{ foot per mile.}
\]

Thus, the river drops about 0.63 foot per mile. (If all distances are converted to feet, then the slope is \(-0.00012\) foot per foot.)

(b) The vertical change in going from Lake Itasca to Memphis is calculated as

\[
\text{Vertical change} = \text{Horizontal change} \times \text{Slope} = 1982 \times -0.63 = -1248.66 \text{ feet.}
\]

Since we started at 1475 feet, the elevation of the river at Memphis is \(1475 - 1248.66 = 226.34\) feet.

(c) The vertical change in going from 1475 feet to 200 feet is \(-1275\) feet.

\[
\begin{aligned}
\text{Vertical change} &= \text{Horizontal change} \times \text{Slope} \\
-1275 &= \text{Horizontal change} \times -0.63 \\
-1275 &= \text{Horizontal change} \\
-0.63 &= \text{Horizontal change} \\
2023.81 &= \text{Horizontal change}. \\
\end{aligned}
\]

**Divide each side by** \(-0.63\).
So we would expect the elevation of the river to be 200 feet when we are 2023.81 miles south of Lake Itasca.

15. A road up a mountain:

(a) The vertical change is \(4960 - 4130 = 830\) feet. The horizontal change is 3 miles. So the slope is

\[
\frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{830}{3} = 276.67 \text{ feet per mile.}
\]

(b) The vertical change in moving 5 miles from the first sign is

\[
\text{Rise} = \text{Run} \times \text{Slope} = 5 \times 276.67 = 1383.35 \text{ feet.}
\]

Since we started at 4130 feet at the first sign, the elevation is \(4130 + 1383.35 = 5513.35\) feet when we are five miles from the first sign.

(c) The vertical change from the first sign to the peak is \(10,300 - 4130 = 6170\) feet.

\[
\frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{6170}{276.67} = \text{Horizontal change} \times 276.67
\]

Divide both sides by 276.67.

Thus the peak is 22.30 horizontal miles away.

16. An underground water source:

(a) Since the limestone layer drops from 220 to 270 feet from 2 to 3 miles, the layer drops 50 feet each mile west of Seiling.

The horizontal change in going from 2 miles to 5 miles west of Seiling is 3 miles. Thus the vertical change is \(3 \times 50 = 150\) feet. Since we started at a 220 foot depth, the limestone should be \(220 + 150 = 370\) feet deep 5 miles west of Seiling.

(b) The vertical change from 220 feet to 290 feet is 70. So the horizontal change is \(\frac{70}{50} = 1.4\) miles. We started 2 miles from Seiling to begin with (where the limestone is 220 feet deep), so we can drill the well at most 3.4 miles west of Seiling.

(c) Now 3 miles west of Seiling the limestone layer is 270 feet deep. If we went one more mile west we would expect the limestone to be \(270 + 50 = 320\) feet deep if it followed a straight line. But it is only 273 feet deep. So the hydrologists were incorrect in their assumption that the limestone followed a straight line.
17. **Earth’s umbra**: We take the positive horizontal axis to pass from the point where the umbra ends through the center of Earth. We find the slope of the upper line shown in the figure by considering the change from the point where the umbra ends to the surface of Earth. We find that the slope is

\[
\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Radius of Earth}}{\text{Distance from Earth to end of umbra}} = \frac{3960}{860,000} \text{ mile per mile.}
\]

Now we find the radius of the umbra at the indicated point by considering the change along the line from the point where the umbra ends to the radius at the indicated point. We use

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change}.
\]

Now the vertical change is the radius of the umbra we are asked to find, and the horizontal change is 860,000 – 239,000 = 621,000. Using the slope calculated above, we find

\[
\text{Radius of the umbra} = \frac{3960}{860,000} \times 621,000 = 2859.49 \text{ miles.}
\]

Thus the radius of the umbra at the indicated point is about 2859 miles. Since the radius of the moon is smaller than this, the moon can fit inside Earth’s umbra. When this happens there is a total lunar eclipse.

18. **The Earth’s penumbra**: We take the positive horizontal axis to pass from the apex through the center of Earth. We find the slope of the upper line shown in the figure by considering the change from the surface of Earth to the point where the penumbra has a radius of 10,000 miles. We find that the slope is

\[
\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{10,000 - \text{Radius of Earth}}{\text{Distance from Earth to the moon}}.
\]

Thus we have

\[
\text{Slope} = \frac{10,000 - 3960}{239,000} = \frac{6040}{239,000} \text{ mile per mile.}
\]

Now we find the distance from Earth to the apex by considering the change from the apex to the surface of Earth. We use

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change}.
\]

Now the vertical change is the radius of Earth, and the horizontal change is the distance we are asked to find. Using the slope calculated above, we find

\[
3960 = \frac{6040}{239,000} \times \text{Distance},
\]

and when we solve this equation we find that the distance is 156,695.36 miles. Thus the apex is about 156,695 miles from Earth.
19. **The umbra of the moon:** We take the positive horizontal axis to pass from the apex (at the center of Earth) to the center of the sun. We find the slope of the line from the apex to the surface of the sun by considering the change from the apex to the surface of the moon. (This can be visualized by replacing Earth by the moon in Figure 3.40.) We find that the slope is

\[
\text{Slope} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Radius of moon}}{\text{Distance from Earth to moon}} = \frac{1100}{239,000} \text{ mile per mile.}
\]

Now we find the radius of the sun by considering the change from the apex to the surface of the sun. We use

\[
\text{Vertical change} = \text{Slope} \times \text{Horizontal change}.
\]

Now the vertical change is the desired radius of the sun, and the horizontal change is the distance from Earth to the sun. Using the slope calculated above, we find

\[
\text{Radius of the sun} = \frac{1100}{239,000} \times 93,498,600 = 430,328.28 \text{ miles.}
\]

Thus the radius of the sun is about 430,328 miles.

### 3.2 LINEAR FUNCTIONS

**E-1. Parallel lines:** We know from the preceding section that parallel lines have the same slope. The line \( y = 3x - 2 \) has slope 3, so we want to find the equation of the line with slope 3 that passes through the point \((3, 3)\). We use the point-slope form:

\[
y - 3 = 3(x - 3)
\]

\[
y - 3 = 3x - 9
\]

\[
y = 3x - 6.
\]

**E-2. Perpendicular lines:** The line \( y = 4x + 1 \) has slope 4, so, by the preceding section, any line perpendicular to the given line has slope \(-\frac{1}{4}\). Hence we want to find the equation of a line with slope \(-\frac{1}{4}\) that passes through the point \((8, 2)\). We use the point-slope form:

\[
y - 2 = -\frac{1}{4}(x - 8)
\]

\[
y - 2 = -\frac{1}{4}x + 2
\]

\[
y = -\frac{1}{4}x + 4.
\]
E-3. Finding equations of lines:

(a) We use the point-slope form:
\[
\begin{align*}
y - 1 &= 3(x - 2) \\
y - 1 &= 3x - 6 \\
y &= 3x - 5.
\end{align*}
\]

(b) We use the point-slope form:
\[
\begin{align*}
y - 1 &= -4(x - 1) \\
y - 1 &= -4x + 4 \\
y &= -4x + 5.
\end{align*}
\]

(c) We use the two-point form. We have
\[
\begin{align*}
y - 2 &= \left(\frac{10 - 2}{5 - 1}\right) (x - 1) \\
y - 2 &= 2x - 2 \\
y &= 2x.
\end{align*}
\]

(d) We use the two-point form. We have
\[
\begin{align*}
y - 1 &= \left(\frac{2 - 1}{-2 - 3}\right) (x - 3) \\
y - 1 &= -\frac{1}{5}x + \frac{3}{5} \\
y &= -\frac{1}{5}x + \frac{8}{5}.
\end{align*}
\]

E-4. Finding the equation of a line: The slope of the line \(3x + 2y = 4\) can be found by putting the line in standard form:
\[
\begin{align*}
3x + 2y &= 4 \\
2y &= -3x + 4 \\
y &= -\frac{3}{2}x + 2.
\end{align*}
\]
Thus the line has slope \(-\frac{3}{2}\). We know from the preceding section that parallel lines have the same slope, so the line we are looking for has slope \(-\frac{3}{2}\) also.

The horizontal intercept of the line \(2x - y = 8\) can be found by putting in \(y = 0\) and solving for \(x\). Thus we have
\[
\begin{align*}
2x - 0 &= 8 \\
2x &= 8 \\
x &= 4.
\end{align*}
\]
Thus the horizontal intercept is 4. This means that the line we are looking for passes through the point $(4, 0)$.

Thus we are looking for a line with slope $-\frac{3}{2}$ that passes through the point $(4, 0)$. We use the point-slope form:

\[
y - 0 = -\frac{3}{2}(x - 4)
\]

\[
y = -\frac{3}{2}x + 6.
\]

Thus the equation is $y = -\frac{3}{2}x + 6$.

S-1. **Slope from two values**: We have that the slope $m$ is:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{19 - 7}{5 - 2} = \frac{12}{3} = 4.
\]

S-2. **Slope from two values**: We have that the slope $m$ is:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{9 - 5}{3 - 8} = \frac{4}{-5} = -0.8.
\]

S-3. **Function value from slope and run**: We have that the slope is $m = 2.7$:

\[
2.7 = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(5) - f(3)}{5 - 3} = \frac{f(5) - 7}{5 - 3} = \frac{f(5) - 7}{2}.
\]

Thus $f(5) - 7 = 2 \times 2.7 = 5.4$, and so $f(5) = 5.4 + 7 = 12.4$.

S-4. **Function value from slope and run**: We have that the slope is $m = 3.1$:

\[
3.1 = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(12) - f(5)}{12 - 5} = \frac{f(12) - 2}{12 - 5} = \frac{f(12) - 2}{7}.
\]

Thus $f(12) - 2 = 7 \times 3.1 = 21.7$, and so $f(12) = 21.7 + 2 = 23.7$.

S-5. **Run from slope and rise**: We have that the slope is $m = -3.4$:

\[
-3.4 = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(x) - f(1)}{x - 1} = \frac{0 - 6}{x - 1} = \frac{-6}{x - 1}.
\]

Thus $-6 = -3.4 \times (x - 1)$, so $x - 1 = \frac{-6}{-3.4} = 1.76$, and therefore $x = 1.76 + 1 = 2.76$.

S-6. **Run from slope and rise**: We have that the slope is $m = 2.6$:

\[
2.6 = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(x) - f(5)}{x - 5} = \frac{0 - (-3)}{x - 5} = \frac{3}{x - 5}.
\]

Thus $3 = 2.6 \times (x - 5)$, so $x - 5 = \frac{3}{2.6} = 1.15$, and therefore $x = 1.15 + 5 = 6.15$. 
S-7. **Linear equation from slope and point:** Since \( f \) is a linear function with slope 4, \( f = 4x + b \) for some \( b \). Now \( f(3) = 5 \), so \( 5 = 4 \times 3 + b \), and therefore \( b = 5 - 4 \times 3 = -7 \). Thus \( f = 4x - 7 \).

S-8. **Linear equation from slope and point:** Since \( f \) is a linear function with slope \(-3\), \( f = -3x + b \) for some \( b \). Now \( f(2) = 8 \), so \( 8 = -3 \times 2 + b \), and therefore \( b = 8 + 3 \times 2 = 14 \). Thus \( f = -3x + 14 \).

S-9. **Linear equation from two points:** We first compute the slope:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(9) - f(4)}{9 - 4} = \frac{2 - 8}{9 - 4} = \frac{-6}{5} = -1.2.
\]

Since \( f \) is a linear function with slope \(-1.2\), \( f = -1.2x + b \) for some \( b \). Now \( f(4) = 8 \), so \( 8 = -1.2 \times 4 + b \), and therefore \( b = 8 + 1.2 \times 4 = 12.8 \). Thus \( f = -1.2x + 12.8 \).

S-10. **Linear equation from two points:** We first compute the slope:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(7) - f(3)}{7 - 3} = \frac{-4 - 5}{7 - 3} = \frac{-9}{4} = -2.25.
\]

Since \( f \) is a linear function with slope \(-2.25\), \( f = -2.25x + b \) for some \( b \). Now \( f(3) = 5 \), so \( 5 = -2.25 \times 3 + b \), and therefore \( b = 5 + 2.25 \times 3 = 11.75 \). Thus \( f = -2.25x + 11.75 \).

S-11. **Linear equation from slope and points:** We are given the slope \( m = -1.7 \) and the initial value \( b = -3.7 \) (because \( f(0) = -3.7 \)). Thus \( f = -1.7x - 3.7 \).

S-12. **Linear equation from slope and points:** Since \( f \) is a linear function with slope \(-1.7\), \( f = -1.7x + b \) for some \( b \). Now \( f(-1) = -3.7 \), so \( -3.7 = -1.7 \times (-1) + b \), and therefore \( b = -3.7 + 1.7 \times (-1) = -5.4 \). Thus \( f = -1.7x - 5.4 \).

S-13. **Linear equation from two points:** We first compute the slope:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(0) - f(-2)}{0 - (-2)} = \frac{3 - 2}{2} = \frac{1}{2} = 0.5.
\]

Now we need the initial value \( b \). Since \( f(0) = 3 \), that value is \( b = 3 \). Thus \( f = 0.5x + 3 \).

S-14. **Linear equation from two points:** We first compute the slope:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{3 - 2}{4} = \frac{1}{4} = 0.25.
\]

Since \( f \) is a linear function with slope \( 0.25 \), \( f = 0.25x + b \) for some \( b \). Now \( f(-2) = 2 \), so \( 2 = 0.25 \times (-2) + b \), and therefore \( b = 2 - 0.25 \times (-2) = 2.5 \). Thus \( f = 0.25x + 2.5 \).
1. Getting Celsius from Fahrenheit:

(a) Water freezes at 0 degrees Celsius, which is 32 degrees Fahrenheit. So $C = 0$ when $F = 32$. Water boils at 100 degrees Celsius, which is 212 degrees Fahrenheit. So $C = 100$ when $F = 212$. These two bits of information allow us to get the slope of the function:

$$\text{Slope} = \frac{\text{Change in } C}{\text{Change in } F} = \frac{100 - 0}{212 - 32} = 0.56.$$  
Thus $C = 0.56F + b$, and we need to find $b$. When $C = 0$, then $F = 32$. This gives $0 = 0.56 \times 32 + b$. Solving for $b$, we get $b = -17.92$. Thus $C = 0.56F - 17.92$.

There are some variations that yield slightly different formulas. The differences are due to the rounding of $\frac{5}{9}$ as 0.56. If the fraction is left as a fraction, then the formula will be $C = \frac{5}{9}F - \frac{160}{9}$, which, to two decimal places, is $C = 0.56F - 17.78$. If the second data point, $C = 100$ when $F = 212$, is used to find $b$ from $C = 0.56F + b$, the resulting calculation will yield $b = -18.72$, giving the formula $C = 0.56F - 18.72$.

(b) The slope we found in Part (a) was 0.56. This means that for every one degree increase in the Fahrenheit temperature, the Celsius temperature will increase by 0.56 degree.

(c) We need to solve $F = 1.8C + 32$ for $C$. We have

$$F = 1.8C + 32$$
$$F - 32 = 1.8C$$
$$\frac{1}{1.8} \times (F - 32) = C$$
$$\frac{1}{1.8}F - \frac{1}{1.8} \times 32 = C$$
$$0.56F - 17.78 = C$$

This is the same as the formula we found in Part (a).

2. A trip to a science fair:

(a) Now

$$\text{Cost} = \text{Cost of admission} + \text{Bus cost}$$
$$C = \$2 \text{ per student} \times \text{Number of students} + \$130$$
$$C = 2n + 130 \text{ dollars.}$$

(b) The slope is 2, and the vertical intercept is 130. The slope indicates that for each additional child we take on the trip the total cost increases by $2. The initial value is $130, and it is the cost of taking the bus itself to the fair.
(c) Here $C(5)$ represents the total cost (in dollars) of the science fair trip if 5 children make the trip. Its value is $C(5) = 2 \times 5 + 130 = 140$ dollars.

(d) We have

\[
2n + 130 = 146 \\
2n = 16 \quad \text{Subtract 130 from each side.} \\
\quad \quad n = 8 \quad \text{Divide both sides by 2.}
\]

The solution of the equation $2n + 130 = 146$ is the number of students we can take if there is $146 to spend. We found that we can take only 8 students for $146.

3. Digitized pictures on a disk drive:

(a) Each additional picture stored increases the total storage space used by 2 megabytes. This means that the change in total storage space used is always the same, 2 megabytes, for a change of 1 in the number of pictures that are stored. Thus the storage space used $S$ is a linear function of the number of pictures stored $n$.

(b) From Part (a), $S$ is a linear function of $n$ with slope 2, since the slope represents the additional storage space used with the addition of 1 picture. The initial value of $S$ is the amount of storage space used if $n = 0$, that is, if no pictures are stored. Since the formatting information, operating system, and applications software use 6000 megabytes, that is the initial value. Thus a formula for $S(n)$ is $S = 2n + 6000$.

(c) The total amount of storage space used on the disk drive if there are 350 pictures stored on the drive is expressed in functional notation as $S(350)$. (Of course, this depends on the choice of function name we made in Part (b).) Its value is $S(350) = 2 \times 350 + 6000 = 6700$ megabytes.

(d) If there are 769,000 megabytes free and the drive holds 800,000 megabytes, then there are $800,000 - 769,000 = 31,000$ megabytes used. To find out how many pictures are stored, we need to solve this equation for $n$:

\[
2n + 6000 = 31,000 \\
\quad 2n = 31,000 - 6000 = 25,000 \quad \text{Subtract 6000 from both sides.} \\
\quad \quad n = \frac{25,000}{2} = 12,500 \quad \text{Divide both sides by 2.}
\]

Thus there are 12,500 digitized pictures stored. There are 769,000 megabytes of room left, and that space can hold $\frac{769,000}{2} = 384,500$ more pictures before the disk drive is full.
4. Speed of sound:

(a) Because $S$ always increases by 1.1 when $T$ increases by 1, $S$ has a constant rate of change and so is a linear function of $T$. The slope is the constant rate of change, namely 1.1 feet per second per degree.

(b) We know that the slope is 1.1, so $S = 1.1T + b$ for some constant $b$. We find $b$ using the fact that $S = 1087.5$ when $T = 32$: We have $1087.5 = 1.1 \times 32 + b$, so $b = 1087.5 - 1.1 \times 32 = 1052.3$. Thus the formula is $S = 1.1T + 1052.3$.

(c) We need to solve $S = 1.1T + 1052.3$ for $T$. We have

\[
S = 1.1T + 1052.3
\]

\[
S - 1052.3 = 1.1T \quad \text{Subtract 1052.3 from each side.}
\]

\[
\frac{S - 1052.3}{1.1} = T \quad \text{Divide both sides by 1.1.}
\]

Thus $T = \frac{S - 1052.3}{1.1}$. Dividing through by 1.1 gives $T = 0.91S - 956.64$.

(d) The slope of $T$ as a linear function of $S$ is 0.91, and this means that an increase in the speed of sound by 1 foot per second corresponds to an increase of 0.91 degree in temperature.

5. Total cost:

(a) Because the variable cost is a constant $20 per widget, for each additional widget produced per month the monthly cost increases by the same amount, $20. This means that $C$ always increases by 20 when $N$ increases by 1. Thus $C$ has a constant rate of change and so is a linear function of $N$. The slope is the constant rate of change, namely 20 dollars per widget. The initial value is the monthly cost when no widgets are manufactured, and this is the amount of the fixed costs, namely $1500. Since the slope is 20 and the initial value is 1500, the formula is $C = 20N + 1500$.

(b) Let $A$ represent the monthly costs (in dollars) for this other manufacturer and $N$ the number of widgets produced in a month. As in Part (a), the slope of this linear function is given by the variable cost, which in this case is 12 dollars per widget. Thus $A = 12N + b$ for some constant $b$. In fact, the constant $b$ is the initial value of the function, and, as in Part (a), this represents the fixed costs. We find $b$ using the fact that $A = 3100$ when $N = 150$: We have $3100 = 12 \times 150 + b$, so $b = 3100 - 12 \times 150 = 1300$. Thus the amount of fixed costs is $1300$.

Another way of finding this is to start with the total cost of $3100 at a production level of $N = 150$ and subtract $12 for each widget produced to account for the
variable cost; the resulting amount of \(3100 - 12 \times 150 = 1300\) dollars is the amount of fixed costs.

(c) Let \(B\) represent the monthly costs (in dollars) for this manufacturer and \(N\) the number of widgets produced in a month. As in Parts (a) and (b), the slope of this linear function is the variable cost, and the initial value of the function represents the fixed costs. We know that \(B = 2700\) when \(N = 100\) and that \(B = 3500\) when \(N = 150\). Thus the slope of \(B\) is given by

\[
\text{Slope} = \frac{\text{Change in } B}{\text{Change in } N} = \frac{3500 - 2700}{150 - 100} = \frac{800}{50} = 16 \text{ dollars per widget.}
\]

Hence the variable cost is $16 per widget. We now know that \(B = 16N + b\), where \(b\) is the initial value (representing the fixed costs). We find \(b\) using the fact that \(B = 2700\) when \(N = 100\): We have \(2700 = 16 \times 100 + b\), so \(b = 2700 - 16 \times 100 = 1100\). Thus the amount of fixed costs is $1100.

6. Total revenue and profit:

(a) Because the selling price is a constant, the total revenue is that price times the number of items. To find the price we divide the total of $2300 for selling 100 widgets by 100 and find that the price is 23 dollars per widget. Thus the formula is \(R = 23N\).

(b) We have \(P = R - C\), and from Part (a) of Exercise 5 we know that \(C = 20N + 1500\). Thus \(P = 23N - (20N + 1500)\), or \(P = 3N - 1500\). The slope of \(P\) is 3 dollars per widget, and this is the difference between the price per widget ($23) and the variable cost ($20). (This makes sense: The extra profit for each extra widget produced is the selling price minus the extra cost to produce that widget.) The initial value of \(P\) is \(-1500\) dollars, and this is the negative of the amount of fixed costs. (This makes sense: When the manufacturer produces no widgets the fixed costs still have to be covered, and the manufacturer has a loss of $1500.)

(c) To find the break-even point we want to find what value of the variable \(N\) gives the function value \(P = 0\). Thus we need to solve the equation \(3N - 1500 = 0\). This can be done either by examining a table of values (or a graph) for the function \(P\) or by solving the linear equation by hand. We take the second approach:

\[
3N - 1500 = 0
\]

\[
3N = 1500
\]

\[
N = \frac{1500}{3} = 500.
\]
Thus the break-even point occurs at a production level of 500 widgets per month.

(d) In the graph below we used a horizontal span of 0 to 1200, as suggested in the exercise, and a vertical span of 0 to 28,000. The point where the graphs cross shows where cost equals revenue, and this is where the profit is zero. Thus it occurs at the break-even point found in Part (c).

7. Slowing down in a curve:

(a) Because \( S \) decreases by 0.746 when \( D \) increases by 1, the slope of \( S \) is \(-0.746 \) mile per hour per degree. The initial value of \( S \) is 46.26 miles per hour because that is the speed on a straight road. Thus the formula is \( S = -0.746D + 46.26 \). Of course, this can also be written as \( S = 46.26 - 0.746D \).

(b) The speed for a road with a curvature of 10 degrees is expressed as \( S(10) \) in functional notation. The value is \( S(10) = -0.746 \times 10 + 46.26 = 38.8 \) miles per hour.

8. Real estate sales:

(a) Let \( N \) be the net income in dollars, and let \( s \) be the total sales in dollars. We know that the net income is \( N = 15,704 \) when sales are \( s = 832,000 \) and that the net income is \( N = 523 \) when sales are \( s = 326,000 \). We use this information to calculate the slope:

\[
\text{Slope} = \frac{\text{Change in net income}}{\text{Change in sales}} = \frac{15,704 - 523}{832,000 - 326,000} = 0.03.
\]

Thus \( N = 0.03s + b \), and we need to find the value of \( b \). Since \( N = 523 \) when \( s = 326,000 \), we have that \( 523 = 0.03 \times 326,000 + b \). Subtracting \( 0.03 \times 326,000 \) from each side and evaluating, we get \( b = -9257 \). The linear equation we need is \( N = 0.03s - 9257 \).

(b) The slope of the line is 0.03, and the vertical intercept is \(-9257 \). We should choose a horizontal span that includes our data points, and so we choose a horizontal span of 0 to 1,000,000 dollars. The table of values below leads us to choose a
vertical span of −15,000 to 25,000 dollars. In the graph below, the horizontal axis corresponds to total sales, and the vertical axis is net income.

(c) If the total sales are 0 then the net income is −9257. This tells us that the fixed monthly cost is $9257.

(d) The slope of the linear function is 0.03. That means for each dollar in sales, the real estate company gets an additional 3 cents. Thus the commission rate is 3%.

(e) The horizontal intercept occurs where \( N = 0 \), and (using the formula) this says

\[
\begin{align*}
0.03s - 9257 &= 0 \\
0.03s &= 9257 \\
s &= 308,566.67
\end{align*}
\]

Add 9257 to each side. Divide both sides by 0.03.

When \( N = 0 \), the agency has a net income of 0 dollars, so it breaks even. Thus the horizontal intercept $308,566.67 (or about $308,570) is the total monthly sales needed for the agency to break even.

9. Currency conversion:

(a) We know we can get \( P = 31 \) pounds for \( D = 83 \) dollars, and we can get \( P = 0 \) pounds for \( D = 0 \) dollars. Thus

\[
\text{Slope} = \frac{\text{Change in pounds}}{\text{Change in dollars}} = \frac{31}{83} = 0.37 \text{ pound per dollar.}
\]

This means each dollar is worth 0.37 pound.

(b) Since we get 0.37 pound per dollar, for 130 dollars we get \( 0.37 \times 130 = 48.10 \) pounds. So the tourist received 48.10 pounds for the 130 dollars.

(c) We want to know how many dollars we get for 12.32 pounds. That is, we want to solve \( 0.37D = 12.32 \) for \( D \). We need only divide each side by 0.37 to get \( D = 33.30 \) dollars. The tourist received $33.30.
10. **Growth in height**:

(a) The boy grows 2 inches each year. That is, the rate of change in height is always 2. Since the rate of change is constant, the boy’s height is a linear function of time, and the slope is 2 inches per year.

(b) Let $H$ denote the boy’s height in inches and $A$ his age in years. We know that $H$ is a linear function of $A$ and that the slope is 2. Thus $H = 2A + b$. We need to find the value of $b$. We also know that the boy’s height is $H = 48$ inches when his age is $A = 9$ years. That means $48 = 2 \times 9 + b$. Subtracting 18 from both sides, we get $b = 30$. We conclude that $H = 2A + 30$ inches.

(c) The initial value of $H$ is $H(0) = 30$.

(d) We know that $H$ is linear for $A$ from 7 to 11 and that the initial value of 30 for $H$ is the vertical intercept of the line. If the actual height for $A$ from 0 to 7 is concave down, then from $A = 0$ to $A = 7$, $H$ is increasing, but at a decreasing rate until age $A = 7$, when the rate of increase of $H$ is 2. Thus, prior to $A = 7$, the rate of increase of $H$ was always greater than 2, and so the curve must have started at $A = 0$ at a smaller value of $H$ than 30. Since newborns are closer to 20 inches long and certainly not 30 inches long, this is quite reasonable.

11. **Adult male height and weight**:

(a) According to this rule of thumb, if a man is 1 inch taller than another, then we expect him to be heavier by 5 pounds. This says that the rate of change in adult male weight is constant, namely 5 pounds per inch, and thus weight is a linear function of height. The slope is the rate of change, 5 pounds per inch.

(b) Let $h$ denote the height in inches and $w$ the weight in pounds. Part (a) tells us that $w$ is a linear function of $h$ with slope 5. Thus $w = 5h + b$. To get the value of $b$, we note that when the weight is $w = 170$ pounds, the height is $h = 70$ inches. Thus $170 = 5 \times 70 + b$, which gives $b = 170 - 5 \times 70 = -180$. We get $w = 5h - 180$.

(c) If the weight is $w = 152$ pounds, then $5h - 180 = 152$. We want to solve this for $h$ to find the height:

\[
5h - 180 = 152 \\
5h = 152 + 180 \\
h = \frac{152 + 180}{5} \\
h = 66.4.
\]

We would expect the man to be 66.4 inches tall.
(d) According to the rule of thumb, if a man is 75 inches tall then his weight is \( w = 5 \times 75 - 180 = 195 \) pounds. The atypical man with a height of 75 inches and a weight of 190 pounds would therefore be light for his height.

12. **Lean body weight in males:**

(a) The slope of \( L \) as a linear function of \( W \) alone (for a fixed value of \( A \)) is 1.08 pounds per pound. This means that their lean body weight increases by 1.08 pounds for every pound of increase in their total weight, assuming that their abdominal circumference remains the same.

(b) The slope of \( L \) as a linear function of \( A \) alone (for a fixed value of \( W \)) is \(-4.14\) pounds per inch. This means that their lean body weight increases by 4.14 pounds for every inch of decrease in their abdominal circumference, assuming that their total weight remains the same.

(c) Using the answers to Parts (a) and (b), we see that an increase of 15 pounds in total weight will increase his lean body weight by \( 15 \times 1.08 = 16.2 \) pounds and that an increase of 2 inches in abdominal circumference will decrease his lean body weight by \( 2 \times 4.14 = 8.28 \) pounds. Thus the net effect will be to increase his body weight by \( 16.2 - 8.28 = 7.92 \) pounds, so his body weight now is \( 144 + 7.92 = 151.92 \) pounds.

13. **Lean body weight in females:** We noted in Exercise 12 that lean body weight in young adult males increases by 1.08 pounds for every pound of increase in total weight, assuming that the abdominal circumference remains the same. For young adult females, the formula shows that the slope of lean body weight as a function of total weight alone (for fixed values of \( R, A, H, \) and \( F \)) is 0.73 pound per pound. This means that their lean body weight increases by only 0.73 pound for every pound of increase in their total weight, assuming that all other factors remain the same.

14. **Horizontal reach of straight streams:**

(a) Because the horizontal factor \( H \) always increases by 6 when \( d \) increases by \( \frac{1}{8} \), the horizontal factor has a constant rate of change and hence is linear.

(b) First we find the slope of \( H \). Because \( H \) always increases by 6 when \( d \) increases by \( \frac{1}{8} \), the slope is given by

\[
\text{Slope} = \frac{\text{Change in } H}{\text{Change in } d} = \frac{6}{\frac{1}{8}} = 48.
\]
We now know that \( H = 48d + b \), where \( b \) is a constant. We find \( b \) using the fact that \( H = 56 \) when \( d = 0.5 \): We have \( 56 = 48 \times 0.5 + b \), so \( b = 56 - 48 \times 0.5 = 32 \). Thus the formula is \( H = 48d + 32 \).

(c) We are given that \( p = 50 \) and \( d = 1.75 \). Using the formula for \( H \) from Part (b), we have \( H = 48 \times 1.75 + 32 \), and thus
\[
S = \sqrt{Hp} = \sqrt{(48 \times 1.75 + 32)50} = 76.16 \text{ feet.}
\]
Hence the horizontal stream will travel 76.16 feet.

(d) We are given that \( d = 1.25 \) and \( p = 70 \). Using the formula for \( H \) from Part (b), we have \( H = 48 \times 1.25 + 32 \), and thus
\[
S = \sqrt{Hp} = \sqrt{(48 \times 1.25 + 32)70} = 80.25 \text{ feet.}
\]
Hence the horizontal stream will travel 80.25 feet. This is greater than the distance of 75 feet, so the stream can reach the fire.

15. Vertical reach of fire hoses:

(a) Because the vertical factor \( V \) always increases by 5 when \( d \) increases by \( \frac{1}{8} \) inch, the vertical factor has a constant rate of change and hence is linear.

(b) First we find the slope of \( V \). Because \( V \) always increases by 5 when \( d \) increases by \( \frac{1}{8} \), the slope is given by
\[
\text{Slope} = \frac{\text{Change in } V}{\text{Change in } d} = \frac{5}{\frac{1}{8}} = 40.
\]
We now know that \( V = 40d + b \), where \( b \) is a constant. We find \( b \) using the fact that \( V = 85 \) when \( d = 0.5 \): We have \( 85 = 40 \times 0.5 + b \), so \( b = 85 - 40 \times 0.5 = 65 \). Thus the formula is \( V = 40d + 65 \).

(c) We are given that \( p = 50 \) and \( d = 1.75 \). Using the formula for \( V \) from Part (b), we have \( V = 40 \times 1.75 + 65 \), and thus
\[
S = \sqrt{Vp} = \sqrt{(40 \times 1.75 + 65)50} = 82.16 \text{ feet.}
\]
Hence the vertical stream will travel 82.16 feet high.

(d) We are given that \( d = 1.25 \) and \( p = 70 \). Using the formula for \( V \) from Part (b), we have \( V = 40 \times 1.25 + 65 \), and thus
\[
S = \sqrt{Vp} = \sqrt{(40 \times 1.25 + 65)70} = 89.72 \text{ feet.}
\]
Hence the horizontal stream will travel 89.72 feet high. This is greater than the height of 60 feet, so the stream can reach the fire.
16. Budget constraints:

(a) If you buy one more pound of apples, they will cost $0.50, so you will have $0.50 less to spend on grapes. Since grapes cost $1 per pound, you must buy a half pound less of grapes.

(b) Now $g$ is the number of pounds of grapes you buy, and $a$ is the number of pounds of apples you buy. According to Part (a), for each additional pound of apples bought, the number of pounds of grapes decreases by 0.5. This means that the rate of change of $g$ as a function of $a$ is constant and equals $-0.5$. Thus $g$ is a linear function of $a$ with a slope of $-0.5$.

(c) If you buy no apples, then you have the full $5 to spend on grapes. Since grapes cost $1 per pound, you will be able to buy 5 pounds of grapes. So the initial value of $g$ is 5.

(d) The slope is $-0.5$ and the initial value is 5, so the formula is $g = -0.5a + 5$ pounds of grapes.

17. More on budget constraints:

(a) If you buy $a$ pounds of apples and each pound costs $0.50, you will have spent $0.50 \times a = 0.50a$ dollars on apples.

(b) If you buy $g$ pounds of grapes and each pound costs $1, you will have spent $1 \times g = g$ dollars on grapes.

(c) Since you will spend a total of $5 on some combination of grapes and apples, then

\[
\text{Money spent on apples} + \text{Money spent on grapes} = $5,\
\]

so $0.50a + g = 5$.

(d) To solve $0.50a + g = 5$ for $g$, subtract $0.50a$ from each side to get $g = -0.50a + 5$, which is the same as the answer for Part (d) from Exercise 16.

18. Traffic signals:

(a) Because the function $n$ increases as $w$ increases, the minimum function value occurs at the smallest relevant value of $w$, namely $w = 0$. Thus the minimum time is the initial value of $n$, which is 3.3 seconds.

(b) Now the slope of $n$ as a function of $w$ is 0.017 second per feet, so if the crossing street for one signal is 10 feet wider than the crossing street for another signal then the wider crossing street should have a $0.017 \times 10 = 0.17$ second longer yellow light.
(c) We have \( n(70) = 3.3 + 0.017 \times 70 = 4.49 \) seconds. This means that the yellow light should be on for 4.49 seconds if the crossing street is 70 feet wide.

(d) We want to find what value of the variable \( w \) gives the function value \( n = 5 \). Thus we need to solve the equation \( 3.3 + 0.017w = 5 \). This can be done either by examining a table of values (or a graph) or by solving the linear equation by hand. We take the second approach:

\[
3.3 + 0.017w = 5 \\
0.017w = 5 - 3.3 \quad \text{Subtract 3.3 from both sides.} \\
w = \frac{5 - 3.3}{0.017} \quad \text{Divide both sides by 0.017.} \\
w = 100.
\]

Thus a crossing-street width of 100 feet would warrant a 5-second yellow light.

19. **Sleeping longer**: If we let \( M \) be the number of hours the man sleeps and \( t \) the number of days since his observation, then \( M \) is a linear function of \( t \), with slope \( \frac{1}{4} \) hour per day (since 15 minutes is \( \frac{1}{4} \) hour) and initial value 8 hours. Thus, \( M = \frac{1}{4}t + 8 \). Since we want to know when the man sleeps 24 hours, we want to find \( t \) so that \( M(t) = 24 \). Thus we want to solve the equation \( \frac{1}{4}t + 8 = 24 \). We find that \( \frac{1}{4}t = 16 \), so \( t = 64 \). Thus, 64 days after his observation the man will sleep 24 hours.

To answer this without using the language of linear functions, note that the exercise is asking how many \( \frac{1}{4} \)-hour segments we need to add to 8 to get 24, i.e., how many \( \frac{1}{4} \)-hour segments there are in 16 hours. Clearly there are 64.

20. **The saros cycle**: We know that at the end of the saros cycle Earth will have rotated one-third revolution beyond its location at the beginning of the cycle. Thus 3 cycles are required for Earth to return to its original location. Now the length of 3 cycles can be found by multiplying 3 by the quantity 18 years plus \( 1\frac{2}{3} \) days, and this is 54 years plus 34 days. Thus a solar eclipse will be viewable three saros cycles, or 54 years and 34 days, after March 7, 1970.

21. **Life on other planets**:

(a) To calculate the most pessimistic value for \( N \) we take the pessimistic value for each of the variables other than \( S \) (because \( N \) is an increasing function of each variable). Thus the most pessimistic value for \( N \) is

\[
N = (2 \times 10^{11}) \times 0.01 \times 0.01 \times 0.01 \times 0.01 \times 10^{-8} = 2 \times 10^{-5}.
\]
Hence the most pessimistic value for $N$ is $2 \times 10^{-5}$, or 0.00002. To calculate the most optimistic value for $N$ we take the optimistic value for each of the variables other than $S$. Thus the most optimistic value for $N$ is

$$N = (2 \times 10^{11}) \times 0.5 \times 1 \times 1 \times 1 \times 10^{-4} = 10,000,000.$$  

Hence the most optimistic value for $N$ is 10,000,000.

(b) Keeping $L$ as a variable and using pessimistic values for each of the other variables (except for $S$) gives

$$N = (2 \times 10^{11}) \times 0.01 \times 0.01 \times 0.01 \times 0.01 \times L = 2000L.$$  

Now we want to find what value of the variable $L$ gives the function value $N = 1$. Thus we need to solve the equation $2000L = 1$. The result is $L = \frac{1}{2000} = 5 \times 10^{-4}$. Thus a value of $L = 5 \times 10^{-4}$, or $L = 0.0005$, will give 1 communicating civilization per galaxy with pessimistic values for the other variables.

(c) Because there is such a difference, even in orders of magnitude, between the pessimistic and optimistic values for the variables and for the function $N$, the formula does not seem helpful in determining if there is life on other planets.

### 3.3 Modeling Data with Linear Functions

E-1. Testing for linearity:

(a) Here are the average rates of change for the data table:

<table>
<thead>
<tr>
<th>Interval</th>
<th>2 to 5</th>
<th>5 to 7</th>
<th>7 to 8</th>
<th>8 to 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rate of change</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

The average rates of change are all 3, so the data are linear, and the slope is 3. Because we know the line passes through $(2, 5)$ we can use the point-slope form to get the equation of the line:

$$y - 5 = 3(x - 2)$$

$$y - 5 = 3x - 6$$

$$y = 3x - 1.$$  

(b) Here are the average rates of change for the data table:

<table>
<thead>
<tr>
<th>Interval</th>
<th>3 to 4</th>
<th>4 to 6</th>
<th>6 to 9</th>
<th>9 to 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rate of change</td>
<td>2</td>
<td>2</td>
<td>$\frac{8}{3}$</td>
<td>2</td>
</tr>
</tbody>
</table>
The average rates of change are not all the same, so the data are not linear.

(c) Here are the average rates of change for the data table:

<table>
<thead>
<tr>
<th>Interval</th>
<th>1 to 3</th>
<th>3 to 7</th>
<th>7 to 9</th>
<th>9 to 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rate of change</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

The average rates of change are all 4, so the data are linear, and the slope is 4. Because we know the line passes through (1, −1) we can use the point-slope form to get the equation of the line:

\[
y - (-1) = 4(x - 1)
\]

\[
y + 1 = 4x - 4
\]

\[
y = 4x - 5.
\]

E-2. **Linear and angular diameter:**

(a) Here are the average rates of change for the data table:

<table>
<thead>
<tr>
<th>Interval</th>
<th>6 to 11</th>
<th>11 to 23</th>
<th>23 to 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rate of change</td>
<td>2.9 × 10⁻⁴</td>
<td>2.9 × 10⁻⁴</td>
<td>2.9 × 10⁻⁴</td>
</tr>
</tbody>
</table>

The average rates of change are all 2.9 × 10⁻⁴ = 0.00029, so the data are linear.

(b) The slope of \( d \) is the average rate of change, which is 0.00029. Because we know the data point \((6, 0.00174)\) we can use the point-slope form to get the equation of the line:

\[
d - 0.00174 = 0.00029(a - 6)
\]

\[
d - 0.00174 = 0.00029a - 0.00174
\]

\[
d = 0.00029a.
\]

(c) We know that the diameter of an object \( k \) kilometers away and showing an angular diameter of \( a \) minutes of arc equals \( k \) times the diameter of an object 1 kilometer away showing an angular diameter of \( a \) minutes of arc. From Part (b) we know that \( d = 0.00029a \) for objects 1 kilometer away. Thus the general relationship is \( d = 0.00029ak \).

(d) We are given that \( k = 384,000 \) and \( a = 31.17 \), and from the formula in Part (c) we obtain \( d = 0.00029 \times 31.17 \times 384,000 = 3471.09 \). Thus the diameter of the moon is about 3471 kilometers.

E-3. **Finding data points:** Using the data points \((2, 5)\) and \((8, 23)\), we find that the slope is \( \frac{23 - 5}{8 - 2} = 3 \). Once we have the slope there are different ways to fill in the missing entries. We will find the equation of the linear model and use this to fill in the blanks. Because
we know the slope, 3, and data point (2, 5) we can use the point-slope form to get the equation of the line:

\[
\begin{align*}
y - 5 &= 3(x - 2) \\
y - 5 &= 3x - 6 \\
y &= 3x - 1.
\end{align*}
\]

Since \(y = 3x - 1\), for \(x = 5\) we have \(y = 3 \times 5 - 1 = 14\). If \(y = 17\) then \(3x - 1 = 17\), and solving for \(x\) gives \(x = 6\). If \(y = 29\) then \(3x - 1 = 29\), and solving for \(x\) gives \(x = 10\). Thus the completed table is

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>5</td>
<td>14</td>
<td>17</td>
<td>23</td>
<td>29</td>
</tr>
</tbody>
</table>

E-4. **Testing for linearity:** Here are the average rates of change for the data table:

<table>
<thead>
<tr>
<th>Interval</th>
<th>(a) to (2a)</th>
<th>(2a) to (5a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average rate of change</td>
<td>(b)</td>
<td>(b)</td>
</tr>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(b)</td>
</tr>
</tbody>
</table>

The average rates of change are all \(\frac{b}{a}\), so the data are linear, and the slope is \(\frac{b}{a}\). Because we know the line passes through \((a, b + a)\) we can use the point-slope form to get the equation of the line:

\[
\begin{align*}
y - (b + a) &= \frac{b}{a}(x - a) \\
y - b - a &= \frac{b}{a}x - b \\
y &= \frac{b}{a}x + a.
\end{align*}
\]

S-1. **Testing data for linearity:** Calculating differences, we see that \(y\) changes by 5 for each change in \(x\) of 2. Thus these data exhibit a constant rate of change and so are linear.

S-2. **Testing data for linearity:** Calculating differences, we see that for each change in \(x\) of 2, the change in \(y\) is 5, then 4, then 4, so the data do not show a constant change in \(y\). The data are not linear.

S-3. **Making a linear model:** The data in Exercise S-1 show a constant rate of change of 5 in \(y\) per change of 2 in \(x\) and therefore the slope is \(m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{5}{2} = 2.5\). We know now that \(y = 2.5x + b\) for some \(b\). Since \(y = 12\) when \(x = 2, 12 = 2.5 \times 2 + b\), so \(b = 12 - 2.5 \times 2 = 7\). The equation is \(y = 2.5x + 7\).

S-4. **Making a linear model:** If we exclude the first point, then the data in Exercise S-2 are linear and show a constant rate of change of 4 in \(y\) per change of 2 in \(x\). Thus the slope is \(m = \frac{\text{Change in } y}{\text{Change in } x} = \frac{4}{2} = 2\). We know now that \(y = 2x + b\) for some \(b\). Since \(y = 17\) when \(x = 4, 17 = 2 \times 4 + b\), so \(b = 17 - 2 \times 4 = 9\). The equation is \(y = 2x + 9\).
S-5. **Graphing discrete data**: The figure below shows a plot of the data in Exercise S-2.

![Graph of discrete data](image)

S-6. **Adding a graph to a data plot**: The figure below shows a plot of the data in Exercise S-2 as well as a graph of the line $y = 2.5x + 7$.

![Graph with line](image)

S-7. **Entering and graphing data**: The entered data are shown in the figure on the left below, and the plot of the data is shown in the figure on the right below.

![Data entry and graph](image)

S-8. **Adding a graph**: The figure below shows the plot of the data from Exercise S-7 together with the graph of $y = -1.7x + 9.7$.

![Combined graph](image)
S-9. **Editing data**: In the figure on the left below, we have entered the squares of the data points in the table from Exercise S-7. The plot of the squares is shown in the figure on the right below.

![Table and Graph](image)

S-10. **Data that are linear**: In the figure below we show a plot of the data from Exercise S-1 along with the linear model \( y = 2.5x + 7 \) we found in Exercise S-3.

![Plot](image)

S-11. **Entering and graphing data**: The entered data are shown in the figure on the left below, and the plot of the data is shown in the figure on the right below.

![Table and Graph](image)
S-12. **Finding and adding linear model**: The data in Exercise S-11 show a constant rate of change of $-6$ in $f$ per change of $3$ in $x$ and therefore the slope is $m = \frac{\text{Change in } f}{\text{Change in } x} = \frac{-6}{3} = -2$. We know now that $f = -2x + b$ for some $b$. Since $f = 2$ when $x = 1$, $2 = -2 \times 1 + b$, so $b = 2 + 2 \times 1 = 4$. The equation is $f = -2x + 4$. The figure below shows the plot of the data from Exercise S-11 together with the graph of $f = -2x + 4$.

1. **Gasoline prices**: Let $t$ be the time in days since January 1, 2004, and let $P$ be the price in cents per gallon.

   (a) The table below shows the differences in successive time periods.

<table>
<thead>
<tr>
<th>Change in $t$</th>
<th>0 to 14</th>
<th>14 to 28</th>
<th>28 to 42</th>
<th>42 to 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in $P$</td>
<td>6.34</td>
<td>6.34</td>
<td>6.34</td>
<td>6.34</td>
</tr>
</tbody>
</table>

   The table of differences shows a constant change of 6.34 over each 14-day period. This shows that the data can be modeled using a linear function.

   (b) In the data display in the left-hand picture below, the window size is automatically selected by the calculator. The horizontal span is $-5.6$ to $61.6$, and the vertical span is $147.69$ to $181.67$. The horizontal axis is time in days, and the vertical axis is the price in cents per gallon.

   (c) The slope of the line is $m = \frac{6.34}{14} = 0.4529$ cents per gallon per day. The initial value is 152 cents per gallon according to the table. Thus the equation is given by $P = 0.4529t + 152$. 
(d) Using the same spans as in Part (b), we obtain the right-hand picture above.

2. Tuition at American private universities:

(a) Let \( d \) be the number of years since 1994 and \( T \) the average tuition in dollars. The difference in the \( d \) values is always 1, and the difference in the \( T \) values is always 866. This shows that the data can be modeled by a linear function with slope 866 dollars per year. Now \( d = 0 \) corresponds to 1994, so (from the data table) the initial value of \( T \) is 13,821, and thus the equation is \( T = 866d + 13,821 \).

(b) The calculator selected a horizontal span from \(-0.4\) to \(4.4\) and a vertical span from \(13,232.12\) to \(17,873.88\). The horizontal axis is years since 1994, and the vertical axis is average tuition.

(c) Now 2003 corresponds to \( d = 9 \), so the predicted average tuition is \( T = 866 \times 9 + 13,821 = 21,615 \) dollars.

3. Tuition at American public universities:

(a) Let \( d \) be the number of years since 1994 and \( T \) the average tuition (in dollars) for public universities. The difference in the \( d \) values is always 1, and the difference in the \( T \) values is always 168. This shows that the data can be modeled by a linear function with slope 168 dollars per year. Now \( d = 0 \) corresponds to 1994, so (from the data table) the initial value of \( T \) is 2816, and thus the equation is \( T = 168d + 2816 \).

(b) The slope for the function from Part (a) is 168 dollars per year.

(c) The slope for the function from Exercise 2 is 866 dollars per year.

(d) Part (b) tells us that the tuition for public institutions increases at a rate of \$168\) per year, while Part (c) tells us that the tuition for private institutions increases at a rate of \$866\) per year. The average tuition at private universities is increasing at a greater rate.
(e) The percentage increase for private universities from 1997 to 1998 is \( \frac{866}{16,419} = 0.053 = 5.3\% \), and the percentage increase for public universities from 1997 to 1998 is \( \frac{168}{3320} = 0.051 = 5.1\% \). The average tuition for private universities had the larger percentage increase from 1997 to 1998.

4. Total cost:

(a) The difference in the \( N \) values is always 50, and the difference in the \( C \) values is always 1750. This shows that the data can be modeled by a linear function with slope \( \frac{1750}{50} = 35 \) dollars per widget. From the solution of Exercise 5 in Section 3.2, we know that the variable cost is the slope, namely 35 dollars per widget. We also know that the amount of the fixed costs is the initial value \( b \) of the function. We find this value as follows: We have \( C = 35N + b \), and we use the fact that \( C = 7900 \) when \( N = 200 \). This gives \( 7900 = 35 \times 200 + b \), and thus \( b = 7900 - 35 \times 200 = 900 \). Hence the amount of the fixed costs is $900.

(b) We want to find what value of \( b \) for the function \( C = 35N + b \) will have the property that \( C = 12,975 \) when \( N = 350 \). This gives \( 12,975 = 35 \times 350 + b \), and thus \( b = 12,975 - 35 \times 350 = 725 \). Hence the amount of the new fixed costs is $725.

(b) Note: Here is another way to solve this part. We want to reduce the fixed costs so as to lower the total cost at a production level of 350 from $13,150 to $12,975, that is, by $175. Then the new fixed costs must be \( 900 - 175 = 725 \) dollars.

(c) Because the variable cost is the slope \( m \), we want to find what value of \( m \) for the function \( C = mN + 900 \) will have the property that \( C = 12,975 \) when \( N = 350 \). This gives \( 12,975 = m \times 350 + 900 \), and thus \( b = \frac{12,975 - 900}{350} = 34.5 \). Hence the new variable cost is 34.50 dollars per widget. Note: Here is another way to solve this part. We want to reduce the variable cost so as to lower the total cost at a production level of 350 from $13,150 to $12,975, that is, by $175. This reduction of total cost must be spread over all 350 widgets, so to accomplish this we must reduce the variable cost by \( \frac{175}{350} = 0.50 \) dollar per widget. Then the new variable cost must be \( 35 - 0.50 = 34.50 \) dollars per widget.

5. Total revenue and profit:

(a) The difference in the \( N \) values is always 50, and the difference in the \( p \) values is always \(-0.50\). This shows that the data can be modeled by a linear function with slope \( \frac{-0.50}{50} = -0.01 \) dollar per widget. Thus we have \( p = -0.01N + b \), and we use the fact that \( p = 43.00 \) when \( N = 200 \) to find \( b \). This gives \( 43.00 = -0.01 \times 200 + b \), and thus \( b = 43.00 + 0.01 \times 200 = 45 \). Hence the model is \( p = -0.01N + 45 \).
(b) Now the total revenue is the price times the number of items, so \( R = pN \). Using the formula from Part (a) gives \( R = (-0.01N + 45)N \). (This can also be written as \( R = -0.01N^2 + 45N \).) This is not a linear function, as can be seen by examining a table of values (or a graph) or by observing that the coefficient of \( N \) is not constant.

(c) We have \( P = R - C \). From Part (a) of Exercise 4 we know that \( C \) has slope 35 and initial value 900, so its formula is \( C = 35N + 900 \). Using the formula for \( R \) from Part (b) of this exercise, we have \( P = (-0.01N + 45)N - (35N + 900) \). (This can also be written as \( P = -0.01N^2 + 10N - 900 \).) This is not a linear function, as can be seen by examining a table of values (or a graph) or by observing that the coefficient of \( N \) is not constant.

6. Dropping rocks on Mars:

(a) The difference in \( V \) for successive seconds is 12.16 feet per second. This tells us that the data can be modeled with a linear equation with slope 12.16 feet per second per second. Since the initial value of \( V \) is 0, the equation is \( V = 16.12t \).

(b) We calculate that \( V(10) = 12.16 \times 10 = 121.6 \) feet per second. This means that 10 seconds after release the velocity of the rock is 121.6 feet per second.

(c) The acceleration due to gravity on Mars is the rate of change in velocity. This is the slope of the line in Part (a), which is 12.16 feet per second per second.

7. The Kelvin temperature scale:

(a) The change in \( F \) is always 36 degrees Fahrenheit for each 20 degree change in \( K \) (the temperature in kelvins). This means that \( F \) is a linear function of \( K \).

(b) The slope is calculated as

\[
\frac{\text{Change in } F}{\text{Change in } K} = \frac{36}{20} = 1.8.
\]

Thus the slope is 1.8 degrees Fahrenheit per kelvin.

(c) The slope of the function is 1.8, so \( F = 1.8K + b \) for some \( b \). To find \( b \), we use one point on the line; for example, when \( K = 200 \), then \( F = -99.67 \). These values give \(-99.67 = 1.8 \times 200 + b\), so \( b = -459.67 \). Thus \( F = 1.8K - 459.67 \).

(d) Now \( F = 98.6 \), and we need to find \( K \). Thus we have to solve the equation \( 1.8K - 459.67 = 98.6 \):

\[
1.8K - 459.67 = 98.6
\]

\[
1.8K = 558.27 \quad \text{Add 459.67 to both sides.}
\]

\[
K = 310.15 \quad \text{Divide both sides by 1.8.}
\]
Thus, body temperature on the Kelvin scale is 310.15 kelvins.

(e) Since the slope of $F$ is 1.8, when temperature increases by one kelvin, the Fahrenheit temperature will increase by 1.8 degrees.

If we solve $F = 1.8K - 459.67$ for $K$ we get $K = \frac{1}{1.8}F + \frac{459.67}{1.8}$, or $K = 0.56F + 255.37$. So a one degree increase in Fahrenheit will increase the temperature by 0.56 kelvin.

(f) If $K = 0$ then $F = 1.8 \times 0 - 459.67 = -459.67$ degrees Fahrenheit.

8. **Further verification of Newton’s second law**: Let $A$ be acceleration, in meters per second per second, and $F$ the force, in newtons.

(a) Subtracting to get the differences gives

<table>
<thead>
<tr>
<th>Change in $A$</th>
<th>3</th>
<th>3</th>
<th>3</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in $F$</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
</tbody>
</table>

Since the change in $F$ is 45 for each change in $A$ of 3, $F$ is a linear function of $A$ with slope $\frac{45}{3} = 15$.

(b) Since the slope is 15, we have $F = 15A + b$. To find $b$, we use the first point given, $A = 8$ meters per second per second when $F = 120$ newtons. Thus $120 = 15 \times 8 + b$, so $b = 0$, and therefore the equation is simply $F = 15A$.

(c) The slope is the mass of 15 kilograms used in the experiment.

(d) The force resulting from an acceleration of 15 meters per second per second is expressed in functional notation as $F(15)$. Its value is $F(15) = 15 \times 15 = 225$ newtons.

(e) Now $F = 15A$, so Force = Mass $\times$ Acceleration, as expected by Newton’s second law of motion.

9. **Market supply**:

(a) By subtracting entries, we see that for each increase of 0.5 in the quantity $S$, there is an increase of 1.05 in price $P$. Thus $P$ is a linear function of $S$ with slope $\frac{1.05}{0.5} = 2.1$. Since the slope is 2.1, we have $P = 2.1S + b$. Now $b$ can be determined using the first data point: $S = 1$ when $P = 1.35$. Substituting, we have $1.35 = 2.1 \times 1.0 + b$, so $b = -0.75$, and therefore $P = 2.1S - 0.75$. 
(b) The given table suggests a horizontal span from 0 to 3 billion bushels of wheat and a vertical span from 0 to 5 dollars per bushel. The horizontal axis is billions of bushels produced, and the vertical axis is price per bushel.

(c) If the price increases, then the wheat suppliers will want to produce more wheat. This means that the market supply curve will be going up to the right, so it is increasing.

(d) If the price \( P \) is $3.90 per bushel, then \( S \) can be determined using the formula from Part (a):
\[
2.1S - 0.75 = 3.90.
\]
Solving (by adding 0.75 to each side, then dividing by 2.1) yields \( S = 2.21 \). Thus suppliers would be willing to produce 2.21 billion bushels in a year for a price of $3.90 per bushel.

10. Market demand:

(a) By subtracting entries we find that, for each increase of 0.5 in \( D \), there is a decrease of 0.30 in \( P \) (a change of \(-0.30\)). Thus \( P \) is a linear function of \( S \) with slope
\[
\frac{\text{Change in } P}{\text{Change in } D} = \frac{-0.30}{0.5} = -0.6.
\]
Since the slope is \(-0.6\), we have \( P = -0.6D + b \). Now \( b \) can be determined using the first data point: \( D = 1 \) when \( P = 2.05 \). Substituting, we have 
\[
2.05 = -0.6 \times 1.0 + b,
\]
so \( b = 2.65 \). Therefore \( P = -0.6D + 2.65 \).

(b) Adding this graph (thick line) to that from Exercise 9 (and using the same window), we get:
(c) As the price of wheat increases, wheat consumers will be willing to purchase less of it. This means that the market demand curve will be doing down as it goes to the right, so it is decreasing.

(d) The point of intersection of the two lines is calculated on the graph above and occurs at $X = 1.26$ and $Y = 1.89$. Since, for both curves, the vertical axis is price, the equilibrium price is $1.89$ per bushel.

11. **Sports car**:

(a) Let $t$ be the time since the car was at rest, in seconds, and $V$ the velocity, in miles per hour. The differences are in the table below:

<table>
<thead>
<tr>
<th>Change in $t$</th>
<th>0.5</th>
<th>0.5</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in $V$</td>
<td>5.9</td>
<td>5.9</td>
<td>5.9</td>
</tr>
</tbody>
</table>

The ratio of change in $V$ to change in $t$ is always $\frac{5.9}{0.5} = 11.8$, so $V$ can be modeled by a linear function of $t$ with slope 11.8.

(b) The slope is 11.8 miles per hour per second. In practical terms, this means that, after every second, the car will be going 11.8 miles per hour faster. This says that the acceleration is constant.

(c) From Part (b), we know that $V = 11.8t + b$ for some $b$. Using the first data point, $t = 2.0$ and $V = 27.9$, we have $27.9 = 11.8 \times 2.0 + b$. Thus $b = 4.3$, and the final formula is $V = 11.8t + 4.3$.

(d) According to the formula in Part (c), the initial velocity is $V(0) = 4.3$ miles per hour. On the other hand, the exercise states that when $t = 0$, the car was at rest, so we would expect the initial velocity to be 0, not 4.3. In practical terms, the formula is valid only after the car has been moving a while since the acceleration is not constant the whole time: acceleration is large initially and decreases afterwards.

(e) We need to find when $V$ is 60. Using the formula, we have $11.8t + 4.3 = 60$, so

\[
t = \frac{60 - 4.3}{11.8} = 4.72 \text{ seconds.}
\]

This car goes from 0 to 60 mph in 4.72 seconds.

12. **High school graduates**: Let $d$ be years since 1985 and $N$ the number of graduating high school students, in millions.

(a) A table of differences is calculated to be:

<table>
<thead>
<tr>
<th>Change in $d$</th>
<th>2</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in $N$</td>
<td>$-0.18$</td>
<td>$-0.18$</td>
<td>$-0.18$</td>
</tr>
</tbody>
</table>
Solution Guide for Chapter 3

The ratio of change in \( N \) to change in \( d \) is always \( \frac{-0.18}{2} = -0.09 \), so this data can be modeled using a linear function with slope \(-0.09\).

(b) The slope is \(-0.09\) million graduating high school students per year. In practical terms, this means that each year there will be 0.09 million, or 90,000, fewer high school students graduating.

(c) To find the initial value, note that 1985 corresponds to \( d = 0 \). Now \( N = 2.83 \) when \( d = 0 \), and thus the formula is \( N = -0.09d + 2.83 \).

(d) The number graduating from high school in 1994 is expressed in functional notation as \( N(9) \) since 1994 is 9 years since 1985. Its value is \( N(9) = -0.09 \times 9 + 2.83 = 2.02 \) million graduating.

13. Later high school graduates: Let \( N \) be the number graduating (in millions) and \( t \) the number of years since 1995.

(a) The difference in the \( t \) values is always 2, and the difference in the \( N \) values is always 0.16. This shows that the data can be modeled by a linear function with slope \( \frac{0.16}{2} = 0.08 \) million graduating per year. The slope means that every year the number graduating increases by 0.08 million (or 80,000).

(b) Now \( t = 0 \) corresponds to 1995, so (from the data table) the initial value of \( N \) is 2.59, and thus the equation is \( N = 0.08t + 2.59 \).

(c) The number graduating from high school in 2002 is expressed in functional notation as \( N(7) \) since 2002 is 7 years since 1995. Its value is \( N(7) = 0.08 \times 7 + 2.59 = 3.15 \) million graduating.

(d) Since 1994 is 1 year before 1995, we use \( t = -1 \) in the formula from Part (b). The value is \( N(-1) = 0.08 \times (-1) + 2.59 = 2.51 \) million graduating. This is much closer to the actual number than the value of 2.02 million graduating given in Part (d) of Exercise 12. The trend in high school graduations was a steady decline from 1985 until the early 1990s followed by a steady increase starting in 1994 or slightly earlier.

14. Tax table: Let \( T \) be the tax and \( I \) the taxable income (both in dollars). The difference in the \( I \) values is always 100. From \( I = 56,300 \) through \( I = 56,700 \) the difference in the \( T \) values is always 15. This shows that the data can be modeled by a linear function with slope \( \frac{15}{100} = 0.15 \) dollar per dollar over that part of the table. The model has the form \( T = 0.15I + b \), and we find the constant \( b \) by using the first entry in the table: \( 7749 = 0.15 \times 56,300 + b \), so \( b = 7749 - 0.15 \times 56,300 = -696 \). Thus the formula from \( I = 56,300 \) through \( I = 56,700 \) is \( T = 0.15I - 696 \).
From $I = 56,800$ through $I = 57,400$ the difference in the $T$ values is always 25. This shows that the data can be modeled by a linear function with slope $\frac{25}{100} = 0.25$ dollar per dollar over that part of the table. The model has the form $T = 0.25I + b$, and we find the constant $b$ by using the entry $I = 56,800$, $T = 7826$ in the table: $7826 = 0.25 \times 56,800 + b$, so $b = 7826 - 0.25 \times 56,800 = -6374$. Thus the formula from $I = 56,800$ through $I = 57,400$ is $T = 0.25I - 6374$.

For each part of the table the slope gives the marginal tax rate for that part—that is, it gives the extra tax owed (in dollars) when the taxable income increases by $1. Thus, for a taxable income between $56,300$ and $56,700$, extra taxable income is taxed at a rate of 15 cents on the dollar. For a taxable income between $56,800$ and $57,400$, extra taxable income is taxed at a rate of 25 cents on the dollar.

15. **Sound speed in oceans:**

(a) Here is a table of differences:

<table>
<thead>
<tr>
<th>Change in $S$</th>
<th>0.6</th>
<th>0.6</th>
<th>0.6</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in $c$</td>
<td>0.72</td>
<td>0.72</td>
<td>0.82</td>
<td>0.62</td>
</tr>
</tbody>
</table>

From this table it is clear that the fourth entry for speed (at the salinity of $S = 36.8$) is in error—it is too high by 0.1. When $S = 36.8$, $c$ should be $1517.62 - 0.1 = 1517.52$. With this choice, all entries are consistent with a linear model.

(b) The linear model has slope

$$\frac{\text{Change in } c}{\text{Change in } S} = \frac{0.72}{0.6} = 1.2 \text{ meters per second per parts per thousand.}$$

The slope means that, at the given depth and temperature, for an increase in salinity of 1 part per thousand the speed of sound increases by 1.2 meters per second.

(c) We use the slope, 1.2, and the last entry in the table. Because

$$\text{Change in } c = \text{Slope} \times \text{Change in } S,$$

we have

$$c(39) - c(37.4) = 1.2 \times (39 - 37.4),$$

and thus

$$c(39) = c(37.4) + 1.2 \times (39 - 37.4) = 1518.24 + 1.2 \times (39 - 37.4) = 1520.16.$$  

This means that the speed of sound is 1520.16 meters per second when the salinity is 39 parts per thousand. **Note:** The value of $c(39)$ can also be found by first using the slope and an entry in the table to find the formula for $c$, namely $c = 1.2S + 1473.36$. 

16. **Focal length:**

(a) The difference in the $F_e$ values is always 0.2, and the difference in the $F_o$ values is always 16. This shows that the data can be modeled by a linear function with slope $\frac{16}{0.2} = 80$ centimeters per centimeter. The model has the form $F_o = 80F_e + b$, and we find the constant $b$ by using the first entry in the table: $24 = 80 \times 0.3 + b$, so $b = 24 - 80 \times 0.3 = 0$. Thus the model is $F_o = 80F_e$.

(b) Because $F_o$ is proportional to $F_e$ with constant of proportionality $M$, the formula is $F_o = MF_e$.

(c) To solve for $M$ in the equation $F_o = MF_e$, we divide both sides by $F_e$ and obtain $M = \frac{F_o}{F_e}$.

(d) To make the fraction in the formula $M = \frac{F_o}{F_e}$ large, we should make the numerator large and the denominator relatively small. Thus the objective lens should have a large focal length and the eyepiece a relatively small focal length.

17. **Measuring the circumference of the Earth:** Because the rate of travel is 100 stades per day, a trip requiring 50 days is $100 \times 50 = 5000$ stades long. This is estimated to be $\frac{1}{50}$ of the Earth’s circumference, so Eratosthenes’ measure of the circumference of the Earth is $5000 \times 50 = 250,000$ stades. This is $250,000 \times 0.104 = 26,000$ miles.

18. **A research project on tax tables:** This is a relatively brief solution.

(a) Let $T$ be the tax and $I$ the taxable income (both in dollars). The difference in the $I$ values is always 100. From $I = 57,300$ through $I = 58,000$ the difference in the $T$ values is always 15. This shows that the data can be modeled by a linear function with slope $\frac{15}{100} = 0.15$ dollar per dollar over that part of the table. The model has the form $T = 0.15I + b$, and we find the constant $b$ by using the first entry in the table: $7884 = 0.15 \times 57,300 + b$, so $b = 7884 - 0.15 \times 57,300 = -711$. Thus the formula from $I = 57,300$ through $I = 58,000$ is $T = 0.15I - 711$.

From $I = 58,100$ through $I = 58,400$ the difference in the $T$ values is always 25. This shows that the data can be modeled by a linear function with slope $\frac{25}{100} = 0.25$ dollar per dollar over that part of the table. The model has the form $T = 0.25I + b$, and we find the constant $b$ by using the entry $I = 58,100$, $T = 8006$ in the table: $8006 = 0.25 \times 58,100 + b$, so $b = 8006 - 0.25 \times 58,100 = -6519$. Thus the formula from $I = 58,100$ through $I = 58,400$ is $T = 0.25I - 6519$.

(b) For both tables the first part is modeled by a linear function with slope 0.15 dollar per dollar, and the second part is modeled by a linear function with slope 0.25
dollar per dollar. For the first table the transition occurs between taxable incomes of $56,700 and $56,800; for the second, the transition occurs between taxable incomes of $58,000 and $58,100. In the later year the transition occurs at higher incomes. The transition level is important because the marginal tax rate increases significantly at this point.

(c) By law the transition level is increased from one year to the next in accordance with the inflation rate. This is an attempt to prevent bracket creep, when taxpayers are bumped into a higher tax bracket by the inflation of incomes even when incomes adjusted for inflation have not risen.

### 3.4 LINEAR REGRESSION

**E-1 Fitting lines:** Answers will vary according to the choice of the estimate line for the federal education expenditures. For example, I might choose a line $E$ with the same slope, 1.275, as Est, but which passes through the third data point (2, 30.4). Such a line has equation $E - 30.4 = 1.275(x - 2)$, so $E = 1.275x + 27.85$. The table below shows the calculations to find the squares of the errors for $E$.

<table>
<thead>
<tr>
<th>Year</th>
<th>Data</th>
<th>Estimate</th>
<th>Error of estimate</th>
<th>Square of error of estimate $(y - E)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>27.0</td>
<td>27.850</td>
<td>-0.850</td>
<td>0.7225</td>
</tr>
<tr>
<td>1</td>
<td>29.0</td>
<td>29.125</td>
<td>-0.125</td>
<td>0.0156</td>
</tr>
<tr>
<td>2</td>
<td>30.4</td>
<td>30.400</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>30.9</td>
<td>31.675</td>
<td>-0.775</td>
<td>0.6006</td>
</tr>
<tr>
<td>4</td>
<td>32.1</td>
<td>32.950</td>
<td>-0.850</td>
<td>0.7225</td>
</tr>
</tbody>
</table>

The sum of the squares of the errors is the sum of the last column, which is 2.0612. Clearly this is not better fit than Est.

**E-2 Calculating regression lines:** In each case, we construct a table to compute $Sx^2$, the sum of the $x$-deviations, and $Sxy$, the sum of the products of the deviations.

(a) The averages are $\bar{x} = \frac{0 + 2 + 4 + 6}{4} = 3$ and $\bar{y} = \frac{1 + 6 + 13 + 20}{4} = 10$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$x$-deviation $x - 3$</th>
<th>$y$-deviation $y - 10$</th>
<th>$x$-deviation squared $(x - 3)^2$</th>
<th>Product of deviations $(x - 3) \times (y - 10)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>-3</td>
<td>-9</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>-1</td>
<td>-4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>3</td>
<td>10</td>
<td>9</td>
<td>30</td>
</tr>
</tbody>
</table>
Thus \( Sx^2 = 9 + 1 + 1 + 9 = 20 \) and \( Sxy = 27 + 4 + 3 + 30 = 64 \), and therefore the regression line is

\[
\text{Reg} = \overline{y} + \frac{Sxy}{Sx^2} (x - \overline{x}) = 10 + \frac{64}{20} (x - 3) = \frac{64}{20} x - 3 \times \frac{64}{20} + 10 = 3.2x + 0.4.
\]

(b) The averages are \( \overline{x} = \frac{0 + 3 + 5 + 9}{4} = 4 \) and \( \overline{y} = \frac{-1 + 6 + 8 + 16}{4} = 7 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )-deviation</th>
<th>( y )-deviation</th>
<th>( x )-deviation squared</th>
<th>Product of deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>-4.25</td>
<td>-8.25</td>
<td>18.0625</td>
<td>35.0625</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>-1.25</td>
<td>-1.25</td>
<td>1.5625</td>
<td>1.5625</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>0.75</td>
<td>0.75</td>
<td>0.5625</td>
<td>0.5625</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
<td>4.75</td>
<td>8.75</td>
<td>22.5625</td>
<td>41.5625</td>
</tr>
</tbody>
</table>

Thus \( Sx^2 = 42.75 \) and \( Sxy = 78.75 \), and therefore the regression line is

\[
\text{Reg} = \overline{y} + \frac{Sxy}{Sx^2} (x - \overline{x}) = \frac{7.25 + 78.75}{42.75} (x - 4.25) = \frac{7.25 \times 78.75}{42.75} + 7.25 = 1.84x - 0.58.
\]

(c) The averages are \( \overline{x} = \frac{1 + 2 + 6 + 7}{4} = 4 \) and \( \overline{y} = \frac{6 + 9 + 23 + 26}{4} = 16 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )-deviation</th>
<th>( y )-deviation</th>
<th>( x )-deviation squared</th>
<th>Product of deviations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>-3</td>
<td>-10</td>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>-2</td>
<td>-7</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>26</td>
<td>3</td>
<td>10</td>
<td>9</td>
<td>30</td>
</tr>
</tbody>
</table>

Thus \( Sx^2 = 26 \) and \( Sxy = 88 \), and therefore the regression line is

\[
\text{Reg} = \overline{y} + \frac{Sxy}{Sx^2} (x - \overline{x}) = 16 + \frac{88}{26} (x - 4) = \frac{88}{26} x - 4 \times \frac{88}{26} + 16 = 3.38x + 2.46.
\]

E-3 Calculating the error: For each data set in Exercise S-2, we form a table that shows the calculations yielding the sum-of-squares error for the regression line \( \text{Reg} \).

(a) Here we use \( \text{Reg} = 3.2x + 0.4 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Regression</th>
<th>Error of regression</th>
<th>Square of error of regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.4</td>
<td>0.6</td>
<td>0.36</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6.8</td>
<td>-0.8</td>
<td>0.64</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
<td>13.2</td>
<td>-0.2</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>20</td>
<td>19.6</td>
<td>0.4</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Thus the sum-of-squares error for the regression line is \( 0.36 + 0.64 + 0.04 + 0.16 = 1.2 \).
(b) Here we use \( \text{Reg} = 1.84x - 0.57 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Regression Reg</th>
<th>Error of regression ( y - \text{Reg} )</th>
<th>Square of error of regression ((y - \text{Reg})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>-0.58</td>
<td>-0.42</td>
<td>0.1764</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>4.95</td>
<td>1.05</td>
<td>1.1025</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>8.63</td>
<td>-0.63</td>
<td>0.3969</td>
</tr>
<tr>
<td>9</td>
<td>16</td>
<td>15.99</td>
<td>0.01</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Thus the sum-of-squares error for the regression line is \( 0.1764 + 1.1025 + 0.3969 + 0.0001 = 1.6759 \), or about 1.68.

(c) Here we use \( \text{Reg} = 3.38x + 2.46 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>Regression Reg</th>
<th>Error of regression ( y - \text{Reg} )</th>
<th>Square of error of regression ((y - \text{Reg})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>5.84</td>
<td>0.16</td>
<td>0.0256</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>9.22</td>
<td>-0.22</td>
<td>0.0484</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>22.74</td>
<td>0.26</td>
<td>0.0676</td>
</tr>
<tr>
<td>7</td>
<td>26</td>
<td>26.12</td>
<td>-0.12</td>
<td>0.0144</td>
</tr>
</tbody>
</table>

Thus the sum-of-squares error for the regression line is \( 0.0256 + 0.0484 + 0.0676 + 0.0144 = 0.156 \).

**E-4 Military expenditures:**

(a) To calculate the regression line \( L \) for military expenditures \( M \) as a function of \( t \), we use a table to show the calculations. Using the data from Example 3.9, we first calculate the averages. The averages are \( \bar{t} = \frac{0 + 1 + 2 + 3 + 4}{5} = 2 \) and \( \bar{M} = \frac{252.7 + 263.3 + 282.0 + 290.4 + 303.6}{5} = 278.4 \).

<table>
<thead>
<tr>
<th>( t )</th>
<th>( M )</th>
<th>( t ) - deviation</th>
<th>( M ) - deviation</th>
<th>( t ) - deviation squared</th>
<th>Product of deviations ((t - \bar{t}) \times (M - \bar{M}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>252.7</td>
<td>-2</td>
<td>-25.7</td>
<td>4</td>
<td>51.4</td>
</tr>
<tr>
<td>1</td>
<td>263.3</td>
<td>-1</td>
<td>-15.1</td>
<td>1</td>
<td>15.1</td>
</tr>
<tr>
<td>2</td>
<td>282.0</td>
<td>0</td>
<td>3.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>290.4</td>
<td>1</td>
<td>12</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>303.6</td>
<td>2</td>
<td>25.2</td>
<td>4</td>
<td>50.4</td>
</tr>
</tbody>
</table>

Thus \( St^2 = 10 \) and \( StM = 128.9 \), and therefore the regression line is

\[
L = \bar{M} + \frac{StM}{St^2} (t-\bar{t}) = 278.4 + \frac{128.9}{10} (t-2) = 12.89t + 278.4 - 2 \times 12.89 = 12.89t + 252.62.
\]

(b) To evaluate the formula for \( L \) from Part (a) at \( t = \bar{t} = 2 \), we calculate \( L(2) = 12.89 \times 2 + 252.62 = 278.4 \), which exactly equals \( \bar{M} \). (In general, it is always true that \( \text{Reg}(\bar{t}) = \bar{y} \).)
E-5 **The meaning of errors:**

(a) If the sum of the squares of the errors of an estimate is 0, then, since each square is non-negative, this would mean that each error is 0. In this case, then, the line would fit the data points exactly.

(b) For each data set and regression line found in Exercise E-2, the errors are found as the fourth column in the tables in Exercise E-3. The sum of each fourth column is zero, allowing for rounding. (In general, it is always true that the sum of the errors of the regression line is 0.)

E-6 **Correlation coefficient:**

(a) Entering the data in a calculator, we have found the regression line and correlation coefficient as shown in the figure below on the left. The regression line is \( y = 2x + 1 \) and the correlation coefficient is \( r = 1 \), indicating an exact fit. The figure on the right below shows a plot of the data together with a graph of the regression line.

(b) Entering the data in a calculator, we have found the regression line and correlation coefficient as shown in the figure below on the left. The regression line is \( y = 2x + 1.4 \) and the correlation coefficient is \( r = 0.99 \), indicating a very close, but not exact, fit. The figure on the right below shows a plot of the data together with a graph of the regression line.
(c) Entering the data in a calculator, we have found the regression line and correlation coefficient as shown in the figure below on the left. The regression line is \( y = 2x + 2.4 \) and the correlation coefficient is \( r = 0.90 \), indicating an approximate fit. The figure on the right below shows a plot of the data together with a graph of the regression line.

S-1. **Slope of regression line**: If the slope of the regression line is positive, then the line is increasing, so we expect overall the data values to be increasing.

S-2. **Meaning of slope of regression line**: If the slope of the regression line for money spent on education as a function of the year is $2300 per year, then this means that as a trend each year the money spent on education increases by about $2300.

S-3. **Meaning of slope of regression line**: If the slope of the regression line for federal agricultural spending is larger than the slope of the regression line for federal spending on research and development, then this means that federal spending on agriculture is increasing faster than federal spending on research and development.

S-4. **Meaning of slope of regression line**: If the slope of the regression line for speed running as a function of length is 2.03 miles per hour per inch, then this means that each additional inch in length adds about 2.03 miles per hour to the running speed.

S-5. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is \( y = -0.26x + 2.54 \). The figure on the right below is the plot of the data with the graph of the regression line added.
S-6. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is \( y = -0.32x + 3.14 \). The figure on the right below is the plot of the data with the graph of the regression line added.

S-7. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is \( y = 1.05x + 2.06 \). The figure on the right below is the plot of the data with the graph of the regression line added.

S-8. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is \( y = 0.21x + 6.93 \). The figure on the right below is the plot of the data with the graph of the regression line added.
S-9. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is $y = -0.32x - 1.59$. The figure on the right below is the plot of the data with the graph of the regression line added.

S-10. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is $y = 3.09x + 3.92$. The figure on the right below is the plot of the data with the graph of the regression line added.

S-11. **Meaning of slope of regression line**: If the slope of the regression line for average life expectancy of children as a function of year of birth is 0.16, then for each increase of 1 in the year of birth the life expectancy increases by 0.16 year, or about 2 months.

S-12. **Plotting data and regression lines**: The figure below on the left is a plot of the data. The regression line is $y = -4.3x + 5.37$. The figure on the right below is the plot of the data with the graph of the regression line added.
1. Is a linear model appropriate?

(a) The plotted points are shown below.

![Plot of data points](image1.png)

The plot of the data appears to be fit better by a curve rather than by a straight line.

(b) We take the variable to be years since 1996. The plotted points are shown below.

![Plot of data points](image2.png)

It looks like the points almost fall on a straight line. It appears reasonable to approximate this data with a straight line.

2. College enrollment: Let \( t \) be the number of years since 1996 and \( E \) the enrollment in public colleges, in millions.

(a) From the left-hand figure below, the equation of the regression line is \( E = 0.13t + 11.1 \). Adding this line to the graph from Exercise 1 gives the right-hand picture below. The horizontal axis is years since 1996, and the vertical axis is enrollment in American public colleges.

![Regression line and data points](image3.png)
(b) The slope of the regression line is 0.13, and this means that every year the enrollments in American public colleges increased by about 0.13 million.

(c) The enrollment in American public colleges in 2002 is $E(6)$ in functional notation, since $t = 6$ corresponds to the year 2002. Using the regression line, we calculate $E(6) = 0.13 \times 6 + 11.1 = 11.88$, or about 11.9, million.

(d) The year 1993 is three years before 1996, so $t = -3$. The regression estimate for 1993 is $E(-3) = 0.13 \times (-3) + 11.1 = 10.71$ million, which is considerably below the actual enrollment of 11.2 million, so it appears that the trend in the late 1990s was not valid as early as 1993.

3. Tourism: Let $t$ be the number of years since 1994 and $T$ the number of tourists, in millions.

(a) A plot of the data points is shown below.

(b) From the left-hand figure below, the equation of the regression line is $T = 1.92t + 18.61$. Adding this line to the plot above gives the right-hand figure below. The horizontal axis is years since 1994, and the vertical axis is millions of tourists.

(c) The slope of the regression line is 1.92, and this means that every year the number of tourists who visited the United States from overseas increases by about 1.92 million.

(d) The number of tourists who visited the United States in 2001 is $T(7)$ in functional notation, since $t = 7$ corresponds to seven years after 1994, that is, the year 2001.
Using the regression line, we calculate \( T(7) = 1.92 \times 7 + 18.61 = 32.05 \) million tourists. This is a good bit higher than the actual number. Surely one factor was the coordinated terrorist attack of September 11, 2001. The general economic slow-down in that time period may have been another factor.

4. **Cable TV**: Let \( t \) be the number of years since 1985 and \( C \) the percentage of American homes with cable TV.

   (a) The plotted points are shown below.

   ![Plot of Cable TV data points]

   (b) From the left-hand picture below, the equation of the regression line is \( C = 2.52t + 45.87 \). Adding this line to the plot above gives the right-hand figure below. The horizontal axis is years since 1985, and the vertical axis is percent of homes with cable TV.

   ![Regression line plot]

   (c) Now 1990 corresponds to \( t = 5 \). Using the equation of the regression line from Part (b), we would predict that \( 2.52 \times 5 + 45.87 = 58.47 \) percent of American homes would have cable TV in 1990.

5. **Long jump**: Let \( t \) be the number of years since 1900 and \( L \) the length of the winning long jump, in meters.

   (a) Using the calculator, we find the equation of the regression line: \( L = 0.034t + 7.197 \).

   (b) In practical terms the meaning of the slope, 0.034 meter per year, of the regression line is that each year the length of the winning long jump increased by an average of 0.034 meter, or about 1.34 inches.
(c) The figure below shows a plot the data points together with a graph of the regression line.

(d) The regression line formula has positive slope and so increases without bound. Surely there is some length that cannot be attained by a long jump, and therefore the regression line is not necessarily a good model of the winning length over a long period of time.

(e) The regression line model gives the value $L(20) = 0.034 \times 20 + 7.197 = 7.877$, or about 7.88, meters, which is over two feet longer than the length of the winning long jump in the 1920 Olympic Games. This result is consistent with our answer to Part (d) above.

6. Driving:

(a) The equation of the regression line expressing $S$ as a linear function of $t$ is $S = 0.23t + 55$.

(b) In practical terms the meaning of the slope, 0.23 mile per hour per second, of the regression line is that for each additional second, our speed increases by 0.23 mile per hour.

(c) Based on the regression line model, our speed will reach 70 miles per hour when $70 = 0.23t + 55$. Solving for $t$, we have $0.23t = 70 - 55 = 15$, and so we will reach 70 miles per hour when $t = \frac{15}{0.23} = 65.22$, or about 65, seconds after starting our observations.
(d) The figure below shows a plot the data points together with a graph of the regression line.

![Graph of data points and regression line]

(e) Looking at the plot in Part (d), we see that the middle three data points are above the line and then the last one is below the line, so it seems that for later times the data points may well stray further below the line. In fact, the plot indicates that the speed is increasing at a decreasing rate. Thus our prediction in Part (c) is likely to give a time earlier than the actual time when our speed reaches 70 miles per hour.

7. **The effect of sampling error on linear regression:**

(a) The plotted points are shown below.

![Plot of plotted points]

(b) From the left-hand figure below, the equation of the regression line is $D = 0.88t + 51.98$. In practical terms, the slope of 0.88 foot per hour means that the flood waters are rising by 0.88 foot each hour.

```
LinReg
y=ax+b
a=0.88
b=51.98
```
(c) Adding the regression line to the plot gives the right-hand figure below. The horizontal axis is time since flooding began, and the vertical axis is depth.

(d) Adding $D = 0.8t + 52$ (thick line) gives the following graph.

So the data does give a close approximation of the depth function, since the two lines are very close. The regression model shows a water level which is a bit too high.

(e) From the depth function $D = 0.8t + 52$, the actual depth of the water after 3 hours is $0.8 \times 3 + 52 = 54.4$ feet.

(f) The regression line equation predicts the depth after 3 hours to be $0.88 \times 3 + 51.98 = 54.62$, or about 54.6, feet.
8. **Gross national product**: Let \( t \) be the number of years since 1998 and \( G \) the gross national product, in trillions of dollars.

(a) From the left-hand figure below, the equation of the regression line is \( G = 0.41t + 8.87 \). In practical terms, the slope of the line being 0.41 tells us that each year the gross national product increases by 0.41 trillion dollars, or about 410 billion dollars.

(b) Plotting both the data points and the regression line gives the right-hand figure below. The horizontal axis is years since 1998, and the vertical axis is gross national product, in trillions of dollars.

(c) Now 2006 corresponds to \( t = 8 \). The equation of the regression line gives a prediction for the gross national product of \( G = 0.41 \times 8 + 8.87 = 12.15 \) trillion dollars. The economist predicted 13 trillion, which is higher, so our information from Part (a) does not support that conclusion. According to the regression line prediction, the gross national product reaches 13 trillion when \( 0.41t + 8.87 = 13 \). Solving for \( t \) gives that \( t = 10.07 \), which roughly corresponds to 2008. So the GNP should about reach 13 trillion dollars by 2008.

9. **Japanese auto sales in the U.S.**: Let \( t \) be the number of years since 1992 and \( J \) the total U.S. sales of Japanese automobiles, in millions.

(a) The data points are plotted below.
It may be reasonable to model this with a linear function, but there is room for argument here. There is enough deviation from a straight line to cast some doubt on whether the use of a linear model would be appropriate. The collection of additional data would be wise.

(b) From the left-hand figure below, the equation of the regression line is \( J = -0.044t + 2.46 \). The slope is -0.044, which means that each year the total US sales of Japanese cars decreases by 0.044 million cars, or about 44,000 cars.

(c) The regression line has been added in the right-hand figure below. The horizontal axis is years since 1992, and the vertical axis is millions of auto sales.

To the eyes of the authors, this picture makes the use of the regression line appear more appropriate than does the appearance of the plotted data alone. Gathering more data still seems advisable.

(d) Now 1997 corresponds to \( t = 5 \), and 1998 corresponds to \( t = 6 \). From the equation of the regression line, we expect \( J = -0.044 \times 5 + 2.46 = 2.24 \) million cars to be sold in 1997 and \( J = -0.044 \times 6 + 2.46 = 2.196 \), or about 2.20, million cars to be sold in 1998. The regression line estimates are higher than the Department of Commerce’s figures in both cases.

10. **Running speed versus length:**

(a) The table indicates that as the length of the animal increases so does the maximum speed. So it is generally true that larger animals run faster.
(b) Looking at the plot below, it does seem that running speed is approximately a linear function of length.

![Plot showing data points almost on a straight line.]

This plot indicates that the data points almost fall on a straight line. It is therefore reasonable to approximate speed as a linear function of length.

(c) From the left-hand figure below, the regression line is $R = 2.03L + 5.09$. The slope indicates that for each additional inch in length an animal should run about 2.03 feet per second faster. The plot of the regression line has been added in the right-hand figure below. The horizontal axis is length, and the vertical axis is speed.

![Regression line equation and plot.]

(d) The data point for the red fox is above the regression line, which means that the red fox runs faster than predicted by the regression line. Since the data point for the cheetah is below the regression line, the cheetah runs more slowly than predicted by the regression line. The red fox is faster for its size.
11. **Antimasonic voting**: Using the notation of \( M \) as the percentage antimasonic voting and \( C \) as the number of church buildings, the statement to be analyzed is “\( M \) depends directly on \( C \),” that is, \( M \) is a linear function of \( C \). The data plot is shown below.

From the left-hand figure below, the equation for the regression line is \( M = 5.89C + 45.55 \); moreover, plotting that equation with the data points gives the right-hand figure. The horizontal axis is number of churches and the vertical axis is percent antimasonic voting.

The figure shows that the regression line fits the data well. So the data does support the premise of the stated direct dependence.

12. **Running ants**: Let \( T \) be the ambient temperature, in degrees Celsius, and \( S \) the ants’ speed, in centimeters per second.

(a) From the figure below, the equation of the regression line for \( S \) is \( S = 0.21T - 2.73 \).
(b) In practical terms, the slope of 0.21 means that for each increase in temperature of one degree Celsius, the ants run 0.21 centimeter per second faster.

(c) Using functional notation, we express the speed at which the ants run when the ambient temperature is 29 degrees Centigrade by \( S(29) \). According to the regression estimate, \( S(29) = 0.21 \times 29 - 2.73 = 3.36 \) centimeters per second.

(d) If the ants are running at 2.5 centimeters per second, then \( S = 2.5 \). Using the equation from Part (a), we have \( 0.21T - 2.73 = 2.5 \). Solving this yields \( T = 24.9 \) degrees Celsius.

13. **Expansion of steam**:

(a) The linear regression model of volume \( V \) as a function of \( T \) is \( V = 12.57T + 7501 \).

(b) If one fire is 100 degrees hotter than another, then \( T \) increases by 100, so \( V \) increases by \( 12.57 \times 100 = 1257 \); that is, the volume of steam produced by 50 gallons of water in the hotter fire is 1257 cubic feet more.

(c) The volume \( V(420) = 12.57 \times 420 + 7501 = 12,780.4 \) cubic feet. In practical terms this answer means that 50 gallons of water applied to a fire of 420 degrees Fahrenheit will produce about 12,780 cubic feet of steam.

(d) If 50 gallons of water expanded to 14,200 cubic feet of steam, then \( V = 14,200 \), and so \( 14,200 = 12.57T + 7501 \). Solving for \( T \), we find that \( 12.57T = 14,200 - 7501 = 6699 \). Thus \( T = \frac{6699}{12.57} = 532.94 \), and therefore the temperature of the fire was 532.94 degrees Fahrenheit.

14. **Technological maturity versus use maturity**: Let \( T \) be technological maturity and let \( U \) be use maturity, each as percents.

(a) Using regression, we find a linear regression model for use maturity as a function of technological maturity: \( U = 1.14T - 12.46 \).

(b) In practical terms the meaning of the slope, 1.14, of the regression line is that a 1 percentage point increase in technological maturity will result in a 1.14 percentage point increase in use maturity. More plainly, an increase of 1 percentage point toward technological perfection of a process means that the process will be used an additional 1.14 percentage point of total reasonable use.

(c) In functional notation, the use maturity of a process that has a technological maturity of 89\%, is \( U(89) \). The value of \( U(89) \), according to the regression model from Part (a), is \( 1.14 \times 89 - 12.46 = 89 \), so the use maturity is also 89\%. 
(d) If a process has a technological maturity of \( T = 73\% \), then the linear model would predict a use maturity of \( U(73) = 1.14 \times 73 - 12.46 = 70.76\% \). Since solvent extraction has a use maturity of only 61\%, it is being used less than would be expected. This information could affect a possible decision to get into the business of selling solvent equipment to industry either as a positive effect (solvent extraction is under-utilized, so there is larger potential market) or a negative effect (solvent extraction is under-utilized, so perhaps it is more difficult to use than previously anticipated).

(e) To construct a linear model for technological maturity as a function of use maturity, we take the equation \( U = 1.14T - 12.46 \) and solve for \( T \):

\[
U = 1.14T - 12.46 \\
U + 12.46 = 1.14T \\
T = \frac{U + 12.46}{1.14}.
\]

This can also be written as \( T = 0.88U + 10.93 \). Alternatively, we could use linear regression directly by interchanging the data columns on our calculator. Regression gives \( T = 0.87U + 11.54 \).

15. Whole crop weight versus rice weight:

(a) Using regression, we find a linear model \( B = 0.29W + 0.01 \) for \( B \) as a function of \( W \).

(b) Comparing the predicted rice weight from the linear model of Part (a) with the data, we see that the third sample, with \( W = 13.7 \) and \( B = 3.7 \) has a significantly lower rice weight than expected from the whole crop weight. This can also be seen by plotting the data with the regression line.

<table>
<thead>
<tr>
<th>( W ) Data</th>
<th>( B ) Data</th>
<th>Predicted ( B = 0.29W + 0.01 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1.8</td>
<td>1.75</td>
</tr>
<tr>
<td>11.1</td>
<td>3.2</td>
<td>3.23</td>
</tr>
<tr>
<td>13.7</td>
<td>3.7</td>
<td>3.98</td>
</tr>
<tr>
<td>14.9</td>
<td>4.3</td>
<td>4.33</td>
</tr>
<tr>
<td>17.6</td>
<td>5.2</td>
<td>5.11</td>
</tr>
</tbody>
</table>

(c) An increase in whole crop weight \( W \) of one ton per hectare is expected to produce an increase in rice weight \( B \) of 0.29 ton per hectare.

16. Rice production in Asia:

(a) An approximate linear model for \( Y \) as a function of \( t \) can be obtained using regression: \( Y = 0.05t + 1.47 \).
(b) The slope, 0.05, of the linear model from Part (a) tells us that each year there is an increase in average rice yield of 0.05 ton per hectare.

(c) According to our model, \(Y(30) = 0.05 \times 30 + 1.47 = 2.97\). In practical terms this means that in 1980 the average rice yield was 2.97 tons per hectare.

(d) The model estimates that for 1990 the rice yield would be \(Y(40) = 0.05 \times 40 + 1.47 = 3.47\) tons per hectare.

(e) Rice production for 2005 is estimated to be \(Y(55) = 0.05 \times 55 + 1.47 = 4.22\) tons per hectare, which is less than the actual rice requirements of 4.92 tons per hectare.

17. **Energy cost of running:**

(a) From the figure below, the equation of regression line for \(E\) in terms of \(v\) is \(E = 0.34v + 0.37\).

(b) The cost of transport is defined to be the slope of the regression line in Part (a), so the cost of transport is 0.34, in units of (unit of \(E\)) per (unit of \(v\)), which is milliliter of oxygen per gram per hour per kilometer per hour.

(c) If a rhea weighs 22,000 grams, then \(W = 22,000\). The formula for \(C\) gives \(C = 8.5 \times 22,000^{-0.40} = 0.16\). The figure found for the cost of transport in Part (b) is much higher than what the general formula would predict. Based on this, the rhea is a much less efficient runner that a typical animal of its size.

(d) If the rhea is at rest, then \(v = 0\). Using \(v = 0\) in the regression equation from Part (a) gives \(E = 0.37\) as an estimate for the oxygen consumption for a rhea at rest. Surely this estimate is higher than the actual oxygen consumption level at rest, since we expect animals to be very inefficient in running at very low speeds.

18. **Laboratory experiment:** Answers will vary. Please refer to the website for details and instructions.

19. **Laboratory experiment:** Answers will vary. Please refer to the website for details and instructions.
20. **Laboratory experiment**: Answers will vary. Please refer to the website for details and instructions.

21. **Research project**: Answers will vary. Please refer to the website for details and instructions.

### 3.5 SYSTEMS OF EQUATIONS

E-1. **Using elimination to solve systems of equations**:

(a) We start with

\[
\begin{align*}
x + y &= 5 \\
x - y &= 1.
\end{align*}
\]

If we add the equations together, since the \(y\)'s cancel out, keep the first equation and replace the second, we have

\[
\begin{align*}
x + y &= 5 \\
2x &= 6.
\end{align*}
\]

Thus \(x = 3\). Plugging \(x = 3\) into the first equation, we have \(3 + y = 5\) and so \(y = 2\).

(b) We start with

\[
\begin{align*}
2x - y &= 0 \\
3x + 2y &= 14.
\end{align*}
\]

If we multiply the first equation by 3 and the second equation by 2, then we have

\[
\begin{align*}
6x - 3y &= 0 \\
6x + 4y &= 28.
\end{align*}
\]

Subtracting the first from the second and, as before, keeping the first, but replacing the second by the difference, we have

\[
\begin{align*}
6x - 3y &= 0 \\
7y &= 28.
\end{align*}
\]

Thus \(y = 4\). Plugging \(y = 4\) into the first equation, we have \(6x - 3 \times 4 = 0\), so \(6x = 12\), and therefore \(x = 2\).
(c) We start with

\[3x + 2y = 6\]
\[4x - 3y = 8.\]

If we multiply the first equation by 4 and the second equation by 3, then we have

\[12x + 8y = 24\]
\[12x - 9y = 24.\]

Subtracting first from the second and, as before, keeping the first, but replacing the second by the difference, we have

\[12x + 8y = 24\]
\[-17y = 0.\]

Thus \(y = 0\). Plugging \(y = 0\) into the first equation, we have \(12x = 24\), and so \(x = 2\).

E-2. **Solving larger systems of equations**:

(a) We start with

\[x + y + z = 3\]
\[2x - y + 2z = 3\]
\[3x + 3y - z = 9.\]

Replacing the second equation with itself minus twice the first equation and replacing the third equation with itself minus three times the first equation, we have

\[x + y + z = 3\]
\[-3y = -3\]
\[-4z = 0.\]

Thus \(z = 0\) and \(y = 1\). Plugging these into the first equation, we get \(x + 1 + 0 = 3\), so \(x = 2\).

(b) We start with

\[x - y - z = -2\]
\[3x - y + 3z = 8\]
\[2x + 2y + z = 6.\]
Replacing the second equation with itself minus three times the first equation and replacing the third equation with itself minus twice the first equation, we have

\[
\begin{align*}
  x - y - z &= -2 \\
  2y + 6z &= 14 \\
  4y + 3z &= 10.
\end{align*}
\]

Now replace the third equation with itself minus twice the second equation:

\[
\begin{align*}
  x - y - z &= -2 \\
  2y + 6z &= 14 \\
  -9z &= -18.
\end{align*}
\]

Thus \( z = 2 \) and, plugging this into the second equation, \( 2y + 6 \times 2 = 14 \), so \( 2y = 2 \) and therefore \( y = 1 \). Finally, plugging the values for \( y \) and \( z \) into the first equation, we have \( x - 1 - 2 = -2 \), so \( x = 1 \).

(c) We start with

\[
\begin{align*}
  x + y + z + w &= 8 \\
  2x - y + z - w &= -1 \\
  3x + y - z + 2w &= 9 \\
  x + 2y + 2z + w &= 12.
\end{align*}
\]

Replacing the second equation with itself minus twice the first equation, replacing the third equation with itself minus three times the first equation, and replacing the fourth equation with itself minus the first equation, we have

\[
\begin{align*}
  x + y + z + w &= 8 \\
  -3y - z - 3w &= -17 \\
  -2y - 4z - w &= -15 \\
  y + z &= 4.
\end{align*}
\]

To help cancel the coefficients of \( y \), multiply the second equation by 2, the third equation by 3, and the fourth equation by 6:

\[
\begin{align*}
  x + y + z + w &= 8 \\
  -6y - 2z - 6w &= -34 \\
  -6y - 12z - 3w &= -45 \\
  6y + 6z &= 24.
\end{align*}
\]
Now replace the third equation with itself minus the second equation and replace the fourth equation with itself plus the second equation:

\[
\begin{align*}
\quad x + y + z + w &= 8 \\
-6y - 2z - 6w &= -34 \\
-10z + 3w &= -11 \\
4z - 6w &= -10.
\end{align*}
\]

Now we multiply the third equation by 2 and the fourth equation by 5:

\[
\begin{align*}
\quad x + y + z + w &= 8 \\
-6y - 2z - 6w &= -34 \\
-20z + 6w &= -22 \\
20z - 30w &= -50.
\end{align*}
\]

Finally we replace the fourth equation by itself plus the third equation:

\[
\begin{align*}
\quad x + y + z + w &= 8 \\
-6y - 2z - 6w &= -34 \\
-20z + 6w &= -22 \\
-24w &= -72.
\end{align*}
\]

Thus \( w = 3 \). Plugging \( w = 3 \) into the third equation, we have \(-20z + 6 \times 3 = -22\), so \(-20z = -40\) and therefore \( z = 2 \). Plugging these values into the second equation, we have \(-6y - 2 \times 2 - 6 \times 3 = -34\), so \(-6y = -12\) and therefore \( y = 2 \). Finally (!), we plug these values into the first equation, yielding \( x + 2 + 2 + 3 = 8 \) and so \( x = 1 \).

E-3. Solving using the augmented matrix: Solving using the augmented matrix is almost identical to what was done in Exercise E-2, only without writing the variables.

(a) We write this system of equations as an augmented matrix:

\[
\begin{pmatrix}
1 & 1 & 1 & 3 \\
2 & -1 & 2 & 3 \\
3 & 3 & -1 & 9
\end{pmatrix}
\]

Eliminating the entries under the upper left-hand corner, we use the indicated row operations:

\[
\begin{pmatrix}
1 & 1 & 1 & 3 \\
0 & -3 & 0 & -3 \\
0 & 0 & -4 & 0
\end{pmatrix} \rightarrow R_2 - 2R_3 \\
\rightarrow R_3 - 3R_1
\]
This matrix corresponds to the system of equations
\[
\begin{align*}
x + y + z &= 3 \\
-3y &= -3 \\
-4z &= 0,
\end{align*}
\]
which can be solved exactly as in Exercise E-2.

(b) We write this system of equations as an augmented matrix:
\[
\begin{pmatrix}
1 & -1 & -1 & -2 \\
3 & -1 & 3 & 8 \\
2 & 2 & 1 & 6
\end{pmatrix}
\]

Eliminating the entries under the upper left-hand corner, we use the indicated row operations:
\[
\begin{pmatrix}
1 & -1 & -1 & -2 \\
0 & 2 & 6 & 14 \\
0 & 4 & 3 & 10
\end{pmatrix}  \quad R_2 - 3R_1 \\
\begin{pmatrix}
1 & -1 & -1 & -2 \\
0 & 2 & 6 & 14 \\
0 & 0 & -9 & -18
\end{pmatrix}  \quad R_3 - 2R_1 \\
\begin{pmatrix}
1 & -1 & -1 & -2 \\
0 & 2 & 6 & 14 \\
0 & 0 & -9 & -18
\end{pmatrix}  \quad R_3 - 2R_2
\]
This matrix corresponds to the system of equations
\[
\begin{align*}
x - y - z &= -2 \\
2y + 6z &= 14 \\
-9z &= -18,
\end{align*}
\]
which can be solved exactly as in Exercise E-2.

(c) We write this system of equations as an augmented matrix:
\[
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
2 & -1 & 1 & -1 & -1 \\
3 & 1 & -1 & 2 & 9 \\
1 & 2 & 2 & 1 & 12
\end{pmatrix}
\]

Eliminating the entries under the upper left-hand corner, we use the indicated row operations:
\[
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -3 & -1 & -3 & -17 \\
0 & -2 & -4 & -1 & -15 \\
0 & 1 & 1 & 0 & 4
\end{pmatrix}  \quad R_2 - 2R_1 \\
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -3 & -1 & -3 & -17 \\
0 & -2 & -4 & -1 & -15 \\
0 & 1 & 1 & 0 & 4
\end{pmatrix}  \quad R_3 - 3R_1 \\
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -3 & -1 & -3 & -17 \\
0 & -2 & -4 & -1 & -15 \\
0 & 1 & 1 & 0 & 4
\end{pmatrix}  \quad R_4 - R_1
\]
We multiply to simplify the elimination in the second column:
\[
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -6 & -2 & -6 & -34 \\
0 & -6 & -12 & -3 & -45 \\
0 & 6 & 6 & 0 & 24
\end{pmatrix}  \quad 2R_2 \\
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -6 & -2 & -6 & -34 \\
0 & -6 & -12 & -3 & -45 \\
0 & 6 & 6 & 0 & 24
\end{pmatrix}  \quad 3R_3 \\
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -6 & -2 & -6 & -34 \\
0 & -6 & -12 & -3 & -45 \\
0 & 6 & 6 & 0 & 24
\end{pmatrix}  \quad 6R_4
And then we eliminate below the $-6$ in the second row:

\[
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -6 & -2 & -6 & -34 \\
0 & 0 & -10 & 3 & -11 \\
0 & 0 & 4 & -6 & -10
\end{pmatrix}
\]

\[R_3 - R_2\]
\[R_4 + R_2\]

Finally we eliminate below the $-10$ in the third row:

\[
\begin{pmatrix}
1 & 1 & 1 & 1 & 8 \\
0 & -6 & -2 & -6 & -34 \\
0 & 0 & -10 & 3 & -11 \\
0 & 0 & 0 & -24 & -72
\end{pmatrix}
\]

\[5R_4 + 2R_3\]

This matrix corresponds to the system of equations

\[
x + y + z + w = 8
\]
\[-6y - 2z - 6w = -34
\]
\[-20z + 6w = -22
\]
\[-24w = -72,
\]

which can be solved exactly as in Exercise E-2.

E-4. **Systems of equations with many solutions:**

(a) We start with

\[
x + y + z = 6
\]
\[2x + y - z = 1
\]
\[4x + 3y + z = 13,
\]

and form its augmented matrix:

\[
\begin{pmatrix}
1 & 1 & 1 & 6 \\
2 & 1 & -1 & 1 \\
4 & 3 & 1 & 13
\end{pmatrix}
\]

We begin to solve by eliminating the terms below the upper left-hand corner:

\[
\begin{pmatrix}
1 & 1 & 1 & 6 \\
0 & -1 & -3 & -11 \\
0 & -1 & -3 & -11
\end{pmatrix}
\]

\[R_2 - 2R_1\]
\[R_3 - 4R_1\]

Eliminating the term below the $-1$ in the second row, we multiply $R_2$ by $-1$ and then we add $R_2$ to $R_3$ to get

\[
\begin{pmatrix}
1 & 1 & 1 & 6 \\
0 & 1 & 3 & 11 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]
(b) We have this system of equations:
\[
\begin{align*}
  x + y + z &= 6 \\
  y + 3z &= 11.
\end{align*}
\]

i. Solving the second equation \(y + 3z = 11\) for \(y\), we obtain \(y = 11 - 3z\).

ii. Solving the first equation \(x + y + z = 6\) for \(x\), we obtain \(x = 6 - y - z\). Using the formula \(y = 11 - 3z\) from Part (i), we get \(x = 6 - (11 - 3z) - z\), or \(x = -5 + 2z\).

(c) We can pick any value for \(z\), use it to calculate \(y = 11 - 3z\) and \(x = -5 + 2z\), and plug them into the original system of equations; they will all be solutions.

E-5. A system of equations with no solution: We start with
\[
\begin{align*}
  x + y + z &= 1 \\
  x + y - z &= 2 \\
  x + y &= 5,
\end{align*}
\]
and form its augmented matrix
\[
\begin{pmatrix}
  1 & 1 & 1 & 1 \\
  1 & 1 & -1 & 2 \\
  1 & 1 & 0 & 5
\end{pmatrix}
\]

Eliminating the terms below the upper left-hand corner, we obtain
\[
\begin{pmatrix}
  1 & 1 & 1 & 1 \\
  0 & 0 & -2 & 1 \\
  0 & 0 & -1 & 4
\end{pmatrix}
\begin{array}{c}
  R_2 - R_1 \\
  R_3 - R_1
\end{array}
\]

Eliminating the term below the \(-2\) in the second row, we obtain
\[
\begin{pmatrix}
  1 & 1 & 1 & 1 \\
  0 & 0 & -2 & 1 \\
  0 & 0 & 0 & 7
\end{pmatrix}
\begin{array}{c}
  2R_3, \text{ then } R_3 - R_2
\end{array}
\]

The third row of this matrix corresponds to the equation \(0x + 0y + 0z = 7\), which is impossible, so the original system has no solutions.

E-6. Determinants:

(a) \[
\begin{vmatrix}
  1 & 2 \\
  3 & 4
\end{vmatrix}
= 1 \times 4 - 2 \times 3 = -2
\]

(b) \[
\begin{vmatrix}
  2 & 2 \\
  6 & 5
\end{vmatrix}
= 2 \times 5 - 2 \times 6 = -2
\]

(c) \[
\begin{vmatrix}
  3 & 1 \\
  3 & 4
\end{vmatrix}
= 3 \times 4 - 1 \times 3 = 9
\]

(d) \[
\begin{vmatrix}
  1 & 2 \\
  1 & 2
\end{vmatrix}
= 1 \times 2 - 2 \times 1 = 0
\]
E-7. Cramer’s rule:

(a) We have

\[
\begin{align*}
  x + 2y &= 5 \\
  x - y &= -1
\end{align*}
\]
so by Cramer’s rule

\[
 x = \frac{5 \cdot (-1) - 2 \cdot (-1)}{1 \cdot (-1) - 2 \cdot 1} = \frac{-3}{-3} = 1
\]

and

\[
 y = \frac{1 \cdot (-1) - 5 \cdot 1}{1 \cdot (-1) - 2 \cdot 1} = \frac{-6}{-3} = 2.
\]

(b) We have

\[
\begin{align*}
  x + y &= 3 \\
  x - y &= 0
\end{align*}
\]
so by Cramer’s rule

\[
 x = \frac{3 \cdot 1 - 1 \cdot 0}{1 \cdot (-1) - 1 \cdot 1} = \frac{-3}{-2} = 1.5
\]

and

\[
 y = \frac{1 \cdot 0 - 3 \cdot 1}{1 \cdot (-1) - 1 \cdot 1} = \frac{-3}{-2} = 1.5.
\]

(c) We have

\[
\begin{align*}
  5x + 3y &= 3 \\
  2x + 3y &= 4
\end{align*}
\]
so by Cramer’s rule

\[
 x = \frac{3 \cdot 3 - 3 \cdot 4}{5 \cdot 3 - 3 \cdot 2} = \frac{-3}{9} = -\frac{1}{3}
\]

and

\[
 y = \frac{5 \cdot 3 - 3 \cdot 2}{5 \cdot 3 - 3 \cdot 2} = \frac{14}{9}.
\]
(d) We have
\[ 6x - y = 5 \]
\[ 3x + 4y = 2 \]
so by Cramer’s rule
\[
x = \frac{\begin{vmatrix} 5 & -1 \\ 2 & 4 \\ 6 & -1 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 6 & 5 \\ 3 & 2 \\ 6 & -1 \\ 3 & 4 \end{vmatrix}} = \frac{5 \times 4 - (-1) \times 2}{6 \times 4 - (-1) \times 3} = \frac{22}{27}
\]
and
\[
y = \frac{\begin{vmatrix} 6 & 5 \\ 3 & 2 \\ 6 & -1 \\ 3 & 4 \end{vmatrix}}{\begin{vmatrix} 6 & 5 \\ 3 & 2 \\ 6 & -1 \\ 3 & 4 \end{vmatrix}} = \frac{6 \times 2 - 5 \times 3}{6 \times 4 - (-1) \times 3} = \frac{-3}{27} = -\frac{1}{9}
\]

S-1. **An explanation**: Each graph consists of pairs of points that make the corresponding equation true. The solution of the system is the point that makes both true, and that is the common intersection point.

S-2. **What is the solution?** Since the graphs do not cross, there is no intersection point, and so there is no solution.

S-3. **Crossing graphs**: We start with
\[ 3x + 4y = 6 \]
\[ 2x - 6y = 5 \]
and solve each equation for \( y \). For the first equation we have \( 3x + 4y = 6 \), so \( 4y = 6 - 3x \) and therefore \( y = \frac{6 - 3x}{4} \). For the second equation we have \( 2x - 6y = 5 \), so \( -6y = 5 - 2x \) and therefore \( y = \frac{5 - 2x}{-6} \). The figure below shows the graph of both \( y = \frac{6 - 3x}{4} \) and \( y = \frac{5 - 2x}{-6} \) using a horizontal span of 0 to 5 and a vertical span of -3 to 3. The intersection point, which is the solution to the system, is at \( x = 2.15 \), \( y = -0.12 \).
S-4. **Crossing graphs:** We start with

\[-7x + 21y = 79\]
\[13x + 17y = 6\]

and solve each equation for \(y\). For the first equation we have \(-7x + 21y = 79\), so \(21y = 79 + 7x\) and therefore \(y = \frac{79 + 7x}{21}\). For the second equation we have \(13x + 17y = 6\), so \(17y = 6 - 13x\) and therefore \(y = \frac{6 - 13x}{17}\). The figure below shows the graph of both \(y = \frac{79 + 7x}{21}\) and \(y = \frac{6 - 13x}{17}\) using a horizontal span of \(-5\) to \(0\) and a vertical span of \(0\) to \(5\). The intersection point, which is the solution to the system, is at \(x = -3.10\), \(y = 2.73\).

![Graph showing the solution](image1)

S-5. **Crossing graphs:** We start with

\[0.7x + 5.3y = 6.6\]
\[5.2x + 2.2y = 1.7\]

and solve each equation for \(y\). For the first equation we have \(0.7x + 5.3y = 6.6\), so \(5.3y = 6.6 - 0.7x\) and therefore \(y = \frac{6.6 - 0.7x}{5.3}\). For the second equation we have \(5.2x + 2.2y = 1.7\), so \(2.2y = 1.7 - 5.2x\) and therefore \(y = \frac{1.7 - 5.2x}{2.2}\). The figure below shows the graph of both \(y = \frac{6.6 - 0.7x}{5.3}\) and \(y = \frac{1.7 - 5.2x}{2.2}\) using a horizontal span of \(-2\) to \(2\) and a vertical span of \(0\) to \(5\). The intersection point, which is the solution to the system, is at \(x = -0.21\), \(y = 1.27\).

![Graph showing the solution](image2)
S-6. **Crossing graphs**: We start with

\[-6.6x - 26.5y = 17.1 \]
\[6.9x + 5.5y = 8.4 \]

and solve each equation for \(y\). For the first equation we have \(-6.6x - 26.5y = 17.1\), so \(-26.5y = 17.1 + 6.6x\) and therefore \(y = \frac{17.1 + 6.6x}{-26.5}\). For the second equation we have \(6.9x + 5.5y = 8.4\), so \(5.5y = 8.4 - 6.9x\) and therefore \(y = \frac{8.4 - 6.9x}{5.5}\). The figure below shows the graph of both \(y = \frac{17.1 + 6.6x}{-26.5}\) and \(y = \frac{8.4 - 6.9x}{5.5}\) using a horizontal span of 0 to 5 and a vertical span of -2 to 2. The intersection point, which is the solution to the system, is at \(x = 2.16\), \(y = -1.18\).

![Graph of the system of equations](image)

S-7. **Hand calculation**: To solve Exercise S-3 by hand calculation, we take the first equation \(3x + 4y = 6\) and solve for \(y\). As shown above for Exercise S-3, this yields \(y = \frac{6 - 3x}{4}\). Now we take the second equation \(2x - 6y = 5\), substitute the expression above for \(y\) and then solve for \(x\):

\[2x - 6\left(\frac{6 - 3x}{4}\right) = 5\]
\[2x - 6\left(\frac{6 - 3x}{4}\right) = 5\]
\[2x - \frac{6}{4}(6 - 3x) = 5\]
\[2x - 6 \times \frac{6}{4} + 3 \times \frac{6}{4}x = 5\]
\[\left(2 + \frac{18}{4}\right)x = 5 + \frac{36}{4} = 5 + 9 = 14\]
\[x = \frac{14}{2 + (9/2)} = 2.15.\]

Now \(y = \frac{6 - 3x}{4} = \frac{6 - 3 \times \left(\frac{14}{2 + (9/2)}\right)}{4} = -0.12\).
S-8. **Hand calculation**: To solve Exercise S-4 by hand calculation, we take the first equation
\[-7x + 21y = 79\] and solve for \(y\). As shown above for Exercise S-4, this yields
\[y = \frac{79 + 7x}{21}.
\]
Now we take the second equation \(13x + 17y = 6\), substitute the expression above for \(y\) and then solve for \(x\):

\[13x + 17y = 6\]
\[13x + 17 \left( \frac{79 + 7x}{21} \right) = 6\]
\[13x + \frac{17(79 + 7x)}{21} = 6\]
\[13x + 79 + \frac{17(79 + 7x)}{21} x = 6\]
\[\left( 13 + \frac{7(79 + 7x)}{21} \right) x = 6 - \frac{79(79 + 7x)}{21}\]
\[x = \frac{6 - \frac{79(79 + 7x)}{21}}{13 + \frac{7(79 + 7x)}{21}} = -3.10.
\]
Now \(y = \frac{79 + 7x}{21} = \frac{79 + 7(-3.10)}{21} = 2.73\).

S-9. **Hand calculation**: To solve Exercise S-5 by hand calculation, we take the first equation
\[0.7x + 5.3y = 6.6\] and solve for \(y\). As shown above for Exercise S-5, this yields \(y = \frac{6.6 - 0.7x}{5.3}\). Now we take the second equation \(5.2x + 2.2y = 1.7\), substitute the expression above for \(y\) and then solve for \(x\):

\[5.2x + 2.2y = 1.7\]
\[5.2x + 2.2 \left( \frac{6.6 - 0.7x}{5.3} \right) = 1.7\]
\[5.2x + \frac{2.2(6.6 - 0.7x)}{5.3} = 1.7\]
\[5.2x + 6.6 - \frac{0.7(2.2x)}{5.3} = 1.7\]
\[\left( 5.2 - 0.7 \frac{2.2}{5.3} \right) x = 1.7 - \frac{6.6}{5.3} \frac{2.2}{5.3}\]
\[x = \frac{1.7 - \frac{6.6}{5.3} \frac{2.2}{5.3}}{5.2 - 0.7 \frac{2.2}{5.3}} = -0.21.
\]
Now \(y = \frac{6.6 - 0.7x}{5.3} = \frac{6.6 - 0.7(-0.21)}{5.3} = 1.27\).

S-10. **Hand calculation**: To solve Exercise S-6 by hand calculation, we take the first equation
\[-6.6x - 26.5y = 17.1\] and solve for \(y\). As shown above for Exercise S-6, this yields \(y = \frac{17.1 + 6.6x}{-26.5}\). Now we take the second equation \(6.9x + 5.5y = 8.4\), substitute the
expression above for $y$ and then solve for $x$:

\[
\begin{align*}
6.9x + 5.5y &= 8.4 \\
6.9x + 5.5\left(\frac{17.1 + 6.6x}{-26.5}\right) &= 8.4 \\
6.9x + \frac{5.5}{-26.5}(17.1 + 6.6x) &= 8.4 \\
6.9x + 17.1\frac{5.5}{-26.5} + 6.6\frac{5.5}{-26.5}x &= 8.4 \\
\left(6.9 + 6.6\frac{5.5}{-26.5}\right)x &= 8.4 - 17.1\frac{5.5}{-26.5} \\
x &= \frac{8.4 - 17.1\frac{5.5}{-26.5}}{6.9 + 6.6\frac{5.5}{-26.5}} = 2.16.
\end{align*}
\]

Now $y = \frac{17.1 + 6.6x}{-26.5} = \frac{17.1 + 6.6 \times 2.16}{-26.5} = -1.18$.

S-11. Crossing graphs: We start with

\[
\begin{align*}
3x - y &= 5 \\
2x + y &= 0
\end{align*}
\]

and solve each equation for $y$. For the first equation we have $3x - y = 5$, so $y = 3x - 5$. For the second equation we have $2x + y = 0$, so $y = -2x$. The figure below shows the graph of both $y = 3x - 5$ and $y = -2x$ using a horizontal span of 0 to 2 and a vertical span of −5 to 1. The intersection point, which is the solution to the system, is at $x = 1$, $y = -2$. 

![Intersection point at (1, -2)](image-url)
S-12. **Hand calculation**: To solve Exercise S-11 by hand calculation, we take the first equation $3x - y = 5$ and solve for $y$. As above for Exercise S-11, this yields $y = 3x - 5$. Now we take the second equation $2x + y = 0$, substitute the expression above for $y$, and then solve for $x$:

\[
2x + y = 0 \\
2x + (3x - 5) = 0 \\
5x - 5 = 0 \\
5x = 5 \\
x = 1.
\]

Now $y = 3x - 5 = 3(1) - 5 = -2$.

1. **A party**: Let $C$ be the number of bags of chips we need to buy and $S$ be the number of sodas we need to buy. We have $36$ to spend, and so

Cost of chips + Cost of sodas = Available money

\[
2C + 0.5S = 36.
\]

Additionally, we know that we will buy 5 times as many sodas as bags of chips:

\[
\text{Sodas} = 5 \times \text{Chips} \\
S = 5C.
\]

Thus, we need to solve the system of equations

\[
2C + 0.5S = 36 \\
S = 5C.
\]

The first step is to solve each equation for one of the variables. Since the second equation is already solved for $S$, we solve the first for $S$ as well:

\[
S = \frac{36 - 2C}{0.5} \\
S = 5C.
\]

We will certainly buy fewer than 20 bags of chips, and so we use a horizontal span of 0 to 20. The following table of values leads us to choose a vertical span of 0 to 90 sodas.

In the right-hand figure below, the thick line is the graph of $S = 5C$. The horizontal axis represents the number of bags of chips, and the vertical axis is the number of sodas.
We see that the graphs cross at \( C = 8 \) and \( S = 40 \). This means that we should buy 8 bags of chips and 40 sodas.

2. Mixing feed:

   (a) In the 30 pounds of alfalfa there are

   \[
   20\% \text{ of } 30 = 0.20 \times 30 = 6 \text{ pounds of protein.}
   \]

   In the 40 pounds of wheat mids there are

   \[
   15\% \text{ of } 40 = 0.15 \times 40 = 6 \text{ pounds of protein.}
   \]

   So altogether there are 12 pounds of protein in the mixture.

   (b) We have

   \[
   \text{Total protein} = 20\% \text{ of alfalfa} + 15\% \text{ of wheat mids}
   \]

   \[
   \text{Total protein} = 0.20a + 0.15w.
   \]

   (c) We want a total of 17% protein in the 1000 pounds of feed. That means the feed should contain \( 0.17 \times 1000 = 170 \) pounds of protein. So from Part (b), we get the equation

   \[
   170 = 0.20a + 0.15w.
   \]

   There are a total of 1000 pounds of cattle feed, and so

   \[
   \text{Total feed} = \text{alfalfa} + \text{wheat mids}
   \]

   \[
   1000 = a + w.
   \]

   Thus, we need to solve the system of equations

   \[
   \begin{align*}
   170 &= 0.20a + 0.15w \\
   1000 &= a + w.
   \end{align*}
   \]
Solving each of these equations for \( w \), we obtain

\[
\begin{align*}
  w &= 1000 - a \\
  w &= \frac{170 - 0.2a}{0.15}.
\end{align*}
\]

If more than half the feed is alfalfa, we will have at least 17.5% protein, and so we will use at most 500 pounds of alfalfa. That is, we want a horizontal span of 0 to 500. The table below leads us to choose a vertical span of 500 to 1000. In the graph below, the thick line corresponds to \( w = 1000 - a \). The horizontal axis is pounds of alfalfa, and the vertical axis is pounds of wheat mids.

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
<td>1133.3</td>
</tr>
<tr>
<td>75</td>
<td>925</td>
<td>933.3</td>
</tr>
<tr>
<td>150</td>
<td>850</td>
<td>733.3</td>
</tr>
<tr>
<td>225</td>
<td>775</td>
<td>533.3</td>
</tr>
<tr>
<td>300</td>
<td>700</td>
<td>333.3</td>
</tr>
<tr>
<td>375</td>
<td>625</td>
<td>133.3</td>
</tr>
<tr>
<td>450</td>
<td>550</td>
<td>33.3</td>
</tr>
</tbody>
</table>

\( X = 0 \)

We see that the graphs intersect at the point \( a = 400 \) and \( w = 600 \). This means we need to mix 400 pounds of alfalfa with 600 pounds of wheat mids to make the cattle feed.

3. **An order for bulbs**: Let \( c \) be the number of crocus bulbs, and let \( d \) be the number of daffodil bulbs. We get our first equation from the fact that the total number of bulbs is 55:

\[
\text{Crocus} + \text{Daffodil} = \text{Total bulbs} \\
\quad c + d = 55.
\]

The second equation we need comes from budget constraints:

\[
\text{Cost of crocuses} + \text{Cost of daffodils} = \text{Total cost} \\
\quad 0.35c + 0.75d = 25.65.
\]

Thus we need to solve the system of equations

\[
\begin{align*}
  c + d &= 55 \\
  0.35c + 0.75d &= 25.65.
\end{align*}
\]
Solving each of these equations for $d$, we get

\[ d = 55 - c \]
\[ d = \frac{25.65 - 0.35c}{0.75}. \]

Since there are certainly no more than 55 crocus bulbs, we use a horizontal span of 0 to 55. From the table below, we choose a vertical span of 0 to 35. In the graph below, the thick line corresponds to $d = \frac{25.65 - 0.35c}{0.75}$. The horizontal axis is the number of crocus bulbs, and the vertical axis is the number of daffodil bulbs.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>55</td>
<td>34.2</td>
</tr>
<tr>
<td>10</td>
<td>54</td>
<td>32.333</td>
</tr>
<tr>
<td>20</td>
<td>53</td>
<td>29.533</td>
</tr>
<tr>
<td>30</td>
<td>52</td>
<td>26.867</td>
</tr>
<tr>
<td>40</td>
<td>51</td>
<td>24.233</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
<td>19.533</td>
</tr>
<tr>
<td>60</td>
<td>50</td>
<td>15.867</td>
</tr>
</tbody>
</table>

We see that the graphs intersect at the point $c = 39$ and $d = 16$. So we should buy 39 crocus bulbs and 16 daffodil bulbs.

4. **American dollars and British pounds**: Let $p$ be the number of British pound notes, and let $d$ be the number of American dollars. We get the first equation from the total number of bills:

\[
\text{Number of pounds} + \text{Number of dollars} = \text{Total bills}
\]

\[ p + d = 17. \]

The second equation comes from the total value (in American dollars) in the wallet:

\[
\text{Value of pounds} + \text{Value of dollars} = \text{Total value}
\]

\[ 2.66p + d = 30.28. \]

Hence we need to solve the system of equations

\[ p + d = 17 \]

\[ 2.66p + d = 30.28. \]

Solving both of these equations for $d$ yields

\[ d = 17 - p \]

\[ d = 30.28 - 2.66p. \]
Since there are 17 bills altogether, we use a horizontal span of 0 to 17. The table of values below leads us to choose a vertical span of 0 to 35. The thick line in the graph below corresponds to \( d = 30.38 - 2.66p \). The horizontal axis is pounds, and the vertical axis is dollars.

<table>
<thead>
<tr>
<th>X</th>
<th>Y1</th>
<th>Y2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>17</td>
<td>30.28</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>22.3</td>
</tr>
<tr>
<td>6</td>
<td>11</td>
<td>14.32</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>6.34</td>
</tr>
<tr>
<td>12</td>
<td>5.6</td>
<td>-1.64</td>
</tr>
<tr>
<td>15</td>
<td>2.6</td>
<td>-9.66</td>
</tr>
<tr>
<td>18</td>
<td>-1</td>
<td>-17.6</td>
</tr>
</tbody>
</table>

We see that the graphs intersect at the point \( p = 8 \) and \( d = 9 \). So we have 8 British pound notes and 9 American dollars.

5. **Population growth**: Let \( f \) be the number of foxes and \( r \) the number of rabbits. Since the rates of change for both foxes and rabbits are constant (33 per year and 53 per year respectively), both are linear functions of time. After \( t \) years, there will be \( f = 255 + 33t \) foxes and \( r = 104 + 53t \) rabbits. We want to know when these values will be the same. That is, we need to find out when \( f = r \), or when \( 255 + 33t = 104 + 53t \). This is a linear equation which we can solve by hand calculation or by graphing. Here is the hand calculation:

\[
255 + 33t = 104 + 53t \\
151 + 33t = 53t \\
151 = 20t \\
7.55 = t
\]

Hence, the number of foxes will be the same as the number of rabbits in 7.55 years. At that time, there will be \( f = 255 + 33 \times 7.55 = 504.15 \) foxes (this is better reported as 504 foxes) and the same number of rabbits.

6. **Teacher salaries**: We begin by calculating the regression lines. For elementary teachers, the regression line is \( E = 0.98t + 35.24 \), where \( t \) is years since 1994 and \( E \) is the salary, in thousands of dollars, of elementary teachers. For secondary teachers, the regression line is \( S = 0.82t + 36.66 \), where \( t \) is years since 1994 and \( S \) is the salary, in thousands of dollars, of secondary teachers. We want to know when these values will be the same. That is, we need to find out when \( E = S \), or when \( 0.98t + 35.24 = 0.82t + 36.66 \). This is
a linear equation which we can solve by hand calculation or by graphing. Here is the hand calculation:

\begin{align*}
0.98t + 35.24 &= 0.82t + 36.66 \\
0.98t &= 0.82t + 36.66 - 35.24 & \text{Subtract 35.24 from each side.} \\
0.98t - 0.82t &= 36.66 - 35.24 & \text{Subtract 0.82 from both sides.} \\
t &= \frac{36.66 - 35.24}{0.98 - 0.82} = 8.88.
\end{align*}

Thus the salaries of elementary and secondary teachers should be equal when \( t = 8.88 \), or \( t = 9 \), rounding to whole numbers. This occurs in the year 1994 + 9 = 2003.

7. **Competition between populations**: The equilibrium point occurs when the per capita growth rate of each population is 0. Thus we have two equations:

\begin{align*}
3(1 - m - n) &= 0 \\
2(1 - 0.7m - 1.1n) &= 0.
\end{align*}

This can solved by hand directly. The first equation is the same as \( 1 - m - n = 0 \), and so \( m = 1 - n \). Substituting this into the second equation, we get \( 2(1 - 0.7(1 - n) - 1.1n) = 0 \). Dividing by 2 and expanding, we get \( 1 - 0.7 + 0.7n - 1.1n = 0 \), so \( 0.3 - 0.4n = 0 \). Thus \( 0.4n = 0.3 \), and so \( n = \frac{0.3}{0.4} = 0.75 \) thousand animals. We can substitute this value to find \( m \). Now \( m = 1 - n = 1 - 0.75 = 0.25 \) thousand animals.

8. **Market supply and demand**: The equilibrium price for wheat is the price \( P \) for which the supply \( S \) equals the demand \( D \). Solving the supply equation for \( S \), we have

\begin{align*}
P &= 2.13S - 0.75 \\
P + 0.75 &= 2.13S \\
\frac{P + 0.75}{2.13} &= S.
\end{align*}

Similarly, for \( D \) we have

\begin{align*}
P &= 2.65 - 0.55D \\
P - 2.65 &= -0.55D \\
\frac{P - 2.65}{-0.55} &= D.
\end{align*}
Now the equilibrium price is when $S = D$, so we have

\[
\begin{align*}
\frac{S}{P + 0.75} &= \frac{D}{P - 2.65} \\
-0.55(P + 0.75) &= 2.13(P - 2.65) \quad \text{Multiplying by } (-0.55) \times 2.13 \\
-0.55P - 0.55 \times 0.75 &= 2.13P - 2.13 \times 2.65 \\
-0.55P - 2.13P &= 0.55 \times 0.75 - 2.13 \times 2.65 \\
P &= \frac{0.55 \times 0.75 - 2.13 \times 2.65}{-0.55 - 2.13} = 1.95.
\end{align*}
\]

So the equilibrium price for wheat is $1.95 per bushel. This can also be found by graphing.

9. **Boron uptake**:

(a) The amount of water-soluble boron $B$ available resulting in the same plant content of boron for Decatur silty clay and Hartxells fine sandy loam is where $33.78 + 37.5B = 31.22 + 71.17B$. You can solve this problem graphically, using a table, or calculating it by hand. By hand, we move terms to the same side to get $33.78 - 31.22 = 71.17B - 37.5B = (71.17 - 37.5)B$. Thus $B = \frac{33.78 - 31.22}{71.17 - 37.5} = 0.08$ part per million.

(b) The value of boron $B$ from Part (a) gives the same plant content $C$ for either soil type. For the Hartxells fine sandy loam, $C$ increases by 71.17 parts per million with each increase of 1 in $B$, whereas for the Decatur silty clay, $C$ only increases by 37.5 parts per million with each increase of 1 in $B$. Thus if $B$ is larger than 0.08 part per million, the Hartxells fine sandy loam will have the larger plant content of boron.
10. **Male and female high school graduates:**

(a) From the left-hand figure below, we see that the regression line is \( Y = -0.126X + 55.72 \), where \( X \) is the number of years since 1960 and \( Y \) is the percentage of male high school graduates enrolled in college.

(b) The right-hand figure below shows that the regression line is \( Y = 0.73X + 39.7 \), where \( X \) is the number of years since 1960 and \( Y \) is the percentage of female high school graduates enrolled in college.

(c) To find out when percentages of males and females are the same, we graph both regression lines and find the intersection point. (This can also be done by hand calculation.) The table of values below leads us to choose a horizontal span of 0 to 30 and a vertical span of 30 to 60. In the graph below, the thick line corresponds to the percentage of females. The horizontal axis is the number of years since 1960, and the vertical axis is the percentage enrolling in college.

<table>
<thead>
<tr>
<th></th>
<th>( X )</th>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>55.72</td>
<td>39.7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>55.09</td>
<td>43.35</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>54.46</td>
<td>47</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>53.83</td>
<td>50.65</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>53.22</td>
<td>54.3</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>52.67</td>
<td>57.95</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>51.94</td>
<td>61.6</td>
</tr>
</tbody>
</table>

We see that the graphs intersect when \( X = 18.71 \) and \( Y = 53.36 \). So we would expect to see the same percentage, 53.36%, of male and female high school graduates entering college at the time 18.72 years after 1960, or in about 1979.
11. **Fahrenheit and Celsius**: When Fahrenheit is exactly twice as much as Celsius, we have \( F = 2C \). That means we need to solve the system of equations

\[
\begin{align*}
F &= \frac{9}{5}C + 32 \\
F &= 2C.
\end{align*}
\]

Since both equations are already solved for \( F \), we need only graph both functions and find the intersection. The table of values below leads us to choose a horizontal span of 150 to 200 and a vertical span of 300 to 400. In the graph below, the thick line corresponds to \( F = 2C \). The horizontal axis is Celsius temperature, and the vertical axis is Fahrenheit temperature.

We see that the graphs intersect when \( C = 160 \) and \( F = 320 \). Thus the temperature we seek is 160 degrees Celsius and 320 degrees Fahrenheit.

12. **A bag of coins**: Let \( D \) be the number of dimes and \( Q \) the number of quarters. Since there are 30 coins, \( D + Q = 30 \). Since the value of the coins is $3.45, \( 0.10D + 0.25Q = 3.45 \).

The first equation can easily be solved for \( Q \), giving \( Q = 30 - D \). Substituting into the second equation, we get

\[
\begin{align*}
0.10D + 0.25Q &= 3.45 \\
0.10D + 0.25(30 - D) &= 3.45 \\
0.10D + 0.25 \times 30 - 0.25D &= 3.45 \\
0.10D - 0.25D &= 3.45 - 0.25 \times 30 \\
D &= \frac{3.45 - 0.25 \times 30}{0.10 - 0.25} = 27.
\end{align*}
\]

Thus the bag contains 27 dimes and 3 quarters.
13. **Parabolic mirrors:**

(a) The point $(2, 4)$ is $(a, a^2)$ where $a = 2$, so the light reflected from $(2, 4)$ follows the line $4 \times 2y + (1 - 4 \times 4)x = 2$, that is, $8y - 15x = 2$. The point $(3, 9)$ is $(a, a^2)$ where $a = 3$, so the light reflected from $(3, 9)$ follows the line $4 \times 3y + (1 - 4 \times 9)x = 3$, that is, $12y - 35x = 3$. The light rays meet at the point where these two equations are both satisfied. Using crossing graphs, for example, we first solve each equation for $y$. The first equation is $8y - 15x = 2$, so $8y = 2 + 15x$ and therefore $y = \frac{2 + 15x}{8}$, while the second equation is $12y - 35x = 3$, so $12y = 3 + 35x$ and therefore $y = \frac{3 + 35x}{12}$. Graphing, we find a common solution at $x = 0$, $y = 0.25$, so the light rays meet at the point $(0, 0.25)$, or $(0, \frac{1}{4})$.

(b) To show that all the reflected light rays pass through the point $(0, 0.25)$, we need only to show that $x = 0$, $y = 0.25$ satisfies the equation $4ay + (1 - 4a^2)x = a$. But plugging in these values, we have $4ay + (1 - 4a^2)x = 4a \times 0.25 + (1 - 4a^2) \times 0 = 4a \times 0.25 + 0 = a \times 1 = a$, so each reflected line does pass through the focal point $(0, 0.25)$.

14. **An interesting system of equations:** To solve the system, we need to graph the linear functions

\[
\begin{align*}
y &= 1 - x \\
y &= 2 - x.
\end{align*}
\]

We have done this below, using a horizontal span of 0 to 5 and a vertical span of $-4$ to 5. The graph below shows that the lines are parallel—they do not intersect. This can be observed without a graph by noticing that the two linear functions have the same slope but different initial values. The system of equations has no solution.
15. **Another interesting system of equations**: Solving both equations for \( y \), we see that we need to graph the two functions

\[
\begin{align*}
    y &= \frac{3 - x}{2} \\
    y &= \frac{6 - 2x}{4}.
\end{align*}
\]

If we do this, we find that both graphs produce the same line, showing that the two functions are in fact identical. If you divide the equation \(-2x - 4y = -6\) by \(-2\), you get the equation \(x + 2y = 3\), which is exactly the first equation. Since the two equations are the same, every point on the line \(x + 2y = 3\) is also on the other line, and hence every such point is a solution of the system of equations.

16. **A system of three equations in three unknowns**:

(a) We get \(z = 3 - 2x + y\).

(b) We get the system of equations

\[
\begin{align*}
    x + y + 2(3 - 2x + y) &= 9 \\
    3x + 2y - (3 - 2x + y) &= 4.
\end{align*}
\]

Solving both of these equations for \(y\), we get the following system:

\[
\begin{align*}
    y &= 1 + x \\
    y &= 7 - 5x.
\end{align*}
\]

(c) If we graph both of the equations in Part (b) using both a horizontal and a vertical span of \(-10\) to \(10\), we get the picture below.

We see that the graphs intersect when \(x = 1\) and \(y = 2\).

(d) From Part (a) we know that \(z = 3 - 2x + y\). Since \(x = 1\) and \(y = 2\), we get \(z = 3 - 2 \times 1 + 2 = 3\). So the solution of the system of equations is \(x = 1, y = 2\) and \(z = 3\).
17. **An application of three equations in three unknowns:** Let \( N \) be the number of nickels, \( D \) the number of dimes, and \( Q \) the number of quarters. Since there are 21 coins in the bag, \( N + D + Q = 21 \). Since there is one more dime than nickel in the bag, \( D = N + 1 \). Since there is $3.35 in the bag, \( 0.05N + 0.10D + 0.25Q = 3.35 \).

This gives three linear equations in the variables \( N, D, \) and \( Q \):

\[
\begin{align*}
N + D + Q &= 21 \\
D &= N + 1 \\
0.05N + 0.10D + 0.25Q &= 3.35.
\end{align*}
\]

The second equation is already solved for \( D \), so we will substitute that into the other two equations. The first equation becomes \( N + (N + 1) + Q = 21 \), or \( 2N + Q + 1 = 21 \). Solving for \( Q \), we get \( Q = 20 - 2N \).

The third equation becomes \( 0.05N + 0.10(N + 1) + 0.25Q = 3.35 \), or \( 0.05N + 0.10N + 0.10 + 0.25Q = 3.35 \). Solving for \( Q \) gives

\[
Q = \frac{3.25 - 0.15N}{0.25}.
\]

Graphing both of these equations using a horizontal span from 0 to 10 nickels and a vertical span from 0 to 30 quarters yields the following picture.

![Graph showing the intersection of two lines at (5, 10)](image)

The graphs intersect when \( N = 5 \) and \( Q = 10 \).

We know that \( D = N + 1 \). Since \( N = 5 \), \( D = 5 + 1 = 6 \). So the solution to the equation is \( N = 5, D = 6, \) and \( Q = 10 \): there are 5 nickels, 6 dimes, and 10 quarters.
Chapter 3 Review Exercises

1. **Drainage pipe slope**: We have \( \text{Slope} = \frac{\text{Rise}}{\text{Run}} \). In this case, the rise is \(-0.5\) foot (since 6 inches make 0.5 foot) for a run of 8 feet. Thus the slope is \( \frac{-0.5}{8} = -0.0625 \) foot per foot.

   If we measure the rise in inches then we find \( \frac{-6}{8} = -0.75 \) inch per foot for the slope.

2. **Height from slope and horizontal distance**: We have

   \[
   \text{Vertical change} = \text{Slope} \times \text{Horizontal change}.
   \]

   In this case, the slope of the ladder is 3.7, while the horizontal distance is 4.4 feet, so the vertical height is \( 3.7 \times 4.4 = 16.28 \) feet.

3. **A ramp into a van**:

   (a) We have \( \text{Slope} = \frac{\text{Rise}}{\text{Run}} \). In this case, the rise is 18 inches, the distance from the van door to the ground, and the run is 4 feet, since that is the horizontal distance from the ramp to the van. Thus the slope is \( \frac{18}{4} = 4.5 \) inches per foot. If we measure the rise in feet then we find \( \frac{1.5}{4} = 0.375 \) foot per foot for the slope.

   (b) We have

   \[
   \text{Rise} = \text{Slope} \times \text{Run}.
   \]

   In this case we take the slope to be 1.2 inches per foot. The rise is still 18 inches, so we have \( 18 = 1.2 \times \text{Run} \). Thus the run is \( \frac{18}{1.2} = 15 \) feet. A larger slope will require a greater distance, so the ramp should rest at least 15 feet from the van.

4. **A reception tent**:

   (a) We have \( \text{Slope} = \frac{\text{Rise}}{\text{Run}} \). The rise from the top of an outside pole to the top of the center pole is \( 25 - 10 = 15 \) feet, and the corresponding run is 30 feet. Thus the slope is \( \frac{15}{30} = 0.5 \) foot per foot.

   (b) We have

   \[
   \text{Rise} = \text{Slope} \times \text{Run}.
   \]

   We know from Part (a) that the slope is 0.5 foot per foot. The run is 5 feet, so the rise is \( 0.5 \times 5 = 2.5 \) feet. Thus the height has increased by 2.5 feet from the outside poles. We add the height of an outside pole to get that the height at this point is \( 2.5 + 10 = 12.5 \) feet.
(c) We have

\[ \text{Rise} = \text{Slope} \times \text{Run}. \]

We know from Part (a) that the slope is 0.5 foot per foot. The rise from the ground
to the top of an outer pole is 10 feet, so we have \(10 = 0.5 \times \text{Run}\). Thus the run is
\[ \frac{10}{0.5} = 20 \text{ feet}. \]
Hence the ropes are attached to the ground 20 feet from the outside poles.

5. **Function value from slope and run**: We have that the slope is \(m = -2.2\), so

\[ -2.2 = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{g(6.8) - g(3.7)}{6.8 - 3.7} = \frac{g(6.8) - 5.1}{6.8 - 3.7} = \frac{g(6.8) - 5.1}{3.1}. \]

Thus \(g(6.8) - 5.1 = 3.1 \times (-2.2)\), and so \(g(6.8) = 5.1 - 3.1 \times (-2.2) = -1.72\).

6. **Lanes on a curved track**:

(a) To find the inner radius of the first lane, we put \(n = 1\) in the formula and find
\(R(1) = \frac{100}{\pi}\). Thus the radius is about 31.83 meters.

(b) The width of a lane is the change in \(R\) when we increase \(n\) by 1, and that is the
slope of the linear function \(R\). From the formula we see that the slope is 1.22. Thus
the width of a lane is 1.22 meters. This can also be found by subtracting successive
values of the radius; for example, \(R(2) - R(1) = 1.22\).

(c) We first find for what value of \(n\) we have \(R(n) = 35\). This means we must solve
the linear equation \(\frac{100}{\pi} + 1.22(n - 1) = 35\) for \(n\). We have

\[
\begin{align*}
100/\pi + 1.22(n - 1) &= 35 \\
1.22(n - 1) &= 35 - 100/\pi \\
n - 1 &= \frac{35 - 100/\pi}{1.22} \\
n &= \frac{35 - 100/\pi}{1.22} + 1 = 3.60.
\end{align*}
\]

Since the number of lanes is a whole number, if you wish to run in a lane with a
radius of at least 35 meters you should pick lane 4 or a higher-numbered lane.

7. **Linear equation from two points**: We first compute the slope:

\[
m = \frac{\text{Change in function}}{\text{Change in variable}} = \frac{g(1.1) - g(-2)}{1.1 - (-2)} = \frac{3.3 - 7.2}{3.1} = \frac{-3.9}{3.1} = -1.258.
\]

(We keep the extra digit for accuracy in the rest of the calculations.) Since \(g\) is a linear
function with slope \(-1.258\), we have \(g = -1.258x + b\) for some \(b\). Now \(g(1.1) = 3.3\), so
\(3.3 = -1.258 \times 1.1 + b\), and therefore \(b = 3.3 + 1.258 \times 1.1 = 4.68\). Thus if we now round
the slope to 2 decimal places we get \(g = -1.26x + 4.68\).
8. **Working on a commission:**

(a) Each additional dollar of total sales increases the income by 5% of $1, or 0.05 dollar. This means that the change in $I$ is always the same, 0.05 dollar, for a change of 1 in $S$. Thus $I$ is a linear function of $S$.

(b) If he sells $1600 in a month then he earns $1000 plus 5% of $1600, or $1000 + 0.05 \times 1600 = 1080$ dollars.

(c) From Part (a), $I$ is a linear function of $S$ with slope 0.05, since the slope represents the additional income for 1 additional dollar in sales. Since $I$ is a linear function with slope 0.05, we have $I = 0.05S + b$ for some $b$. Now $I(1600) = 1080$ by Part (b), so $1080 = 0.05 \times 1600 + b$, and therefore $b = 1080 - 0.05 \times 1600 = 1000$. Thus $I = 0.05S + 1000$.

(d) We want to find the value of $S$ so that $I = 1350$. That means we need to solve the equation $0.05S + 1000 = 1350$ for $S$. We find:

\[
0.05S + 1000 = 1350 \\
0.05S = 1350 - 1000 \\
S = \frac{1350 - 1000}{0.05} = 7000.
\]

Thus his monthly sales must be $7000.

9. **Testing data for linearity:**

(a) There is a constant change of 0.3 in $x$ and a constant change of $-0.06$ in $f$. Thus these data exhibit a constant rate of change and so are linear.

(b) From Part (a) we know that the data show a constant rate of change of $-0.6$ in $f$ per change of 0.3 in $x$. Hence the slope is $m = \frac{\text{Change in } f}{\text{Change in } x} = \frac{-0.6}{0.3} = -2$.

We know now that $f = -2x + b$ for some $b$. Since $f = 8$ when $x = 3$, we have $8 = -2 \times 3 + b$, so $b = 8 + 2 \times 3 = 14$. The formula is $f = -2x + 14$.

10. **An epidemic:**

(a) There is a constant change of 5 in $d$ and a constant change of 6 in $T$. Thus these data exhibit a constant rate of change and so are linear.

(b) The slope represents the number of new cases per day.

(c) The slope is $m = \frac{\text{Change in } T}{\text{Change in } d} = \frac{6}{5} = 1.2$.

(d) Now $T$ is a linear function with slope 1.2, and the initial value is 35 (from the first entry in the table). Thus the formula is $T = 1.2d + 35$. 

(e) To predict the total number of diagnosed flu cases after 17 days we put \( d = 17 \) in the formula from Part (d): \( T = 1.2 \times 17 + 35 = 55.4 \). Thus we expect 55 flu cases to be diagnosed after 17 days.

11. Testing and plotting linear data:

(a) There is a constant change of 2 in \( x \) and a constant change of 0.24 in \( f \). Thus these data exhibit a constant rate of change and so are linear.

(b) The slope is \( m = \frac{\text{Change in } f}{\text{Change in } x} = \frac{0.24}{2} = 0.12 \). We know now that \( f = 0.12x + b \) for some \( b \). Since \( f = -1.12 \) when \( x = -1 \), we have \(-1.12 = 0.12 \times (-1) + b \), so \( b = -1.12 + 0.12 = -1 \). The formula is \( f = 0.12x - 1 \).

(c) The figure below shows the plot of the data together with the graph of \( f = 0.12x - 1 \).

12. Marginal tax rate:

(a) There is a constant change of $50 in the taxable income and a constant change of $14 in the tax due. Thus these data exhibit a constant rate of change and so are linear.

(b) From Part (a) the rate of change in tax due as a function of taxable income is \( \frac{14}{50} = 0.28 \) dollar per dollar. Thus the additional tax due on each dollar is $0.28.

(c) From the table the tax due on a taxable income of $97,000 is $21,913. If the taxable income increases by $1000 to $98,000, by Part (b) the tax due will increase by \( 0.28 \times 1000 = 280 \) dollars. Thus the tax due on a taxable income of $98,000 is $21,913 + 280 = $22,193 dollars.

(d) Let \( T \) denote the tax due, in dollars, on an income of \( A \) dollars over $97,000. The slope of \( T \) is 0.28 by Part (b). The initial value of \( T \) is the tax due on 0 dollars over $97,000, that is, on $97,000. That tax is $21,913. Thus the formula is \( T = 0.28A + 21,913 \).
13. **Plotting data and regression lines:**

(a) The figure below on the left is a plot of the data.

(b) The regression line is \( y = -16.3x + 37.19 \).

(c) The figure on the right below is the plot of the data with the regression line added.

14. **Meaning of slope of regression line:** If the slope of the regression line for number of births per thousand to unmarried 18- to 19-year-old women as a function of year is \(-1.13\), then for each year that passes the number of births to unmarried 18- to 19-year-old women decreases by about 1.16 births per thousand.

15. **Life expectancy:**

(a) Let \( E \) be life expectancy in years and \( t \) the time in years since 1996. Using regression gives the model \( E = 0.2t + 76.2 \).

(b) The figure below is the plot of the data with the regression line added.

(c) The slope of the regression line is 0.2, and this means that for each increase of 1 in the year of birth the life expectancy increases by 0.2 year, or about \( 2 \frac{1}{2} \) months.
(d) Now 2005 corresponds to $t = 9$, so using the regression line we predict the life expectancy to be $0.2 \times 9 + 76.2 = 78$ years.

(e) Now 1580 corresponds to $t = -416$, so from the regression line we expect the life expectancy to be $0.2 \times (-416) + 76.2 = -7$ years. This is nonsensical! The year 2300 corresponds to $t = 304$, so from the regression line we predict the life expectancy to be $0.2 \times 304 + 76.2 = 137$ years. This is unreasonable. In fact, both of these predictions are far out of the range of applicability of the regression line formula.

16. **XYZ Corporation stock prices:**

(a) Let $P$ be the stock price in dollars and $t$ the time in months since January, 2005. Using regression gives the model $P = 0.547t + 43.608$.

(b) The figure below is the plot of the data with the regression line added.

\[ \text{Plot of data with regression line added.} \]

(c) The slope is 0.547 dollar per month, and this means that the stock price increases by about $0.55$ each month.

(d) Now January, 2006 corresponds to $t = 12$, so using the regression line we predict the stock price to be $0.547 \times 12 + 43.608 = 50.172$, or about $50.17$. Also January, 2007 corresponds to $t = 24$, so using the regression line we predict the stock price to be $0.547 \times 24 + 43.608 = 56.736$, or about $56.74$. 

\[ \text{Graph showing the stock price predictions.} \]
17. **Crossing graphs**: We start with

\[
\begin{align*}
4x - 2y &= 9 \\
x + y &= 0
\end{align*}
\]

and solve each equation for \(y\). For the first equation we have \(4x - 2y = 9\), so \(2y = 4x - 9\) and therefore \(y = \frac{4x - 9}{2}\). For the second equation we have \(x + y = 0\), so \(y = -x\). The figure below shows the graph of both \(y = \frac{4x - 9}{2}\) and \(y = -x\) using a horizontal span of 0 to 2 and a vertical span of −5 to 0. The intersection point, which is the solution to the system, is at \(x = 1.5\), \(y = -1.5\).

18. **Hand calculation**: To solve Exercise 17 by hand calculation, we take the second equation \(x + y = 0\) and solve for \(y\). As above for Exercise 17, this yields \(y = -x\). Now we take the first equation \(4x - 2y = 9\), substitute the expression above for \(y\), and then solve for \(x\):

\[
\begin{align*}
4x - 2y &= 9 \\
4x - 2(-x) &= 9 \\
6x &= 9 \\
x &= \frac{9}{6} = 1.5.
\end{align*}
\]

Now \(y = -x = -1.5\).
19. **Bills**: Let $T$ be the number of twenty-dollar bills and $F$ the number of fifty-dollar bills. Since the total value is $400, we have $20T + 50F = 400$. Since there are 11 bills, we have $T + F = 11$. The second equation can easily be solved for $F$, giving $F = 11 - T$. Substituting into the first equation, we get

\[
20T + 50(11 - T) = 400
\]

\[
20T + 50 	imes 11 - 50T = 400
\]

\[
-30T = 400 - 50 	imes 11
\]

\[
T = \frac{400 - 50 	imes 11}{-30} = 5.
\]

Thus there are 5 twenty-dollar bills and 6 fifty-dollar bills.

20. **Mixing paint**: Let $B$ be the number of gallons of blue paint and $Y$ the number of gallons of yellow paint. Since the total is 10 gallons, we have $B + Y = 10$. Since there is 3 times as much yellow paint as blue paint, we have $Y = 3B$. We insert the formula for $Y$ from the second equation into the first:

\[
B + Y = 10
\]

\[
B + 3B = 10
\]

\[
4B = 10
\]

\[
B = \frac{10}{4} = 2.5.
\]

Thus you should use 2.5 gallons of blue paint and 7.5 gallons of yellow paint.